Plasma Physics Reports, Vol. 28, No. 2, 2002, pp. 125–139. Translated from Fizika Plazmy, Vol. 28, No. 2, 2002, pp. 144–160.

PARTICLE ACCELERATION IN PLASMA

Charged Particle Acceleration by an Intense Ultrashort Electromagnetic Pulse Excited in a Plasma by Laser Radiation or by Relativistic Electron Bunches

V. A. Balakirev, V. I. Karas', and I. V. Karas'

National Science Center Kharkov Institute of Physics and Technology, ul. Akademicheskaya 1, Kharkov, 310108 Ukraine
e-mail: karas@kipt.kharkov.ua

Received September 27, 2001

Abstract—A review is given of theoretical and experimental investigations and numerical simulations of the generation of intense electromagnetic fields in accelerators based on collective methods of charged particle acceleration at rates two or three orders of magnitude higher than those in classical resonance accelerators. The conditions are studied under which the excitation of accelerating fields by relativistic electron bunches or intense laser radiation in a plasma is most efficient. Such factors as parametric and modulational processes, the generation of a quasistatic magnetic field, and the acceleration of plasma electrons and ions are investigated in order to determine the optimum conditions for the most efficient acceleration of the driven charged-particle bunches. © *2002 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

One of the most promising methods for collective acceleration [1–9] is a plasma-based scheme for particle acceleration by space-charge waves [3]. There are many papers devoted to the development of such schemes [10–77]. The most important of these are the following: paper [3], in which this method was first proposed by Fainberg; paper [4], in which Tajima and Dawson suggested new efficient methods for exciting plasma waves by laser light [plasma beat-wave accelerator (PBWA) and laser wake-field accelerator (LWFA) schemes] and analyzed some important relevant problems of particle acceleration; and paper [10] by Chen *et al.*, who proposed to excite plasma waves by a short electron bunch or a periodic train of electron bunches [plasma wake-field accelerator (PWFA)]. An important point here is that it was suggested to accelerate particles by wake plasma waves. Note that the question of the excitation of an electromagnetic field by an electron bunch in a plasma has already been discussed in the literature [69–74]. The self-consistent dynamics of rectangular electron bunches in a plasma has been examined in many papers (see, e.g., [14, 36, 78] and the literature cited therein). Theoretical investigations [15–18] and experimental works [19–22] (see also [48]) have also substantially contributed to the development of acceleration schemes based on laser-driven plasma waves, and papers [12, 13, 79–82] made significant contributions to research on the excitation of wake plasma waves by electron bunches in PWFA schemes. In recent years, the wake-field acceleration method has been substantially modified: a new version—Self-Modulated Wake-Field Acceleration (SMWFA), which is based on the self-modulation of a laser pulse—was proposed in [15–20, 83–85] (see also [52]). The most impressive results on plasma acceleration of charged particles were obtained in experimental studies on LWFA [19–22], in which the accelerating fields at short distances were as strong as $1.5–20 \times 10^8$ V/cm and particles were accelerated to energies of 100–300 MeV over distances of about 1 cm. Thus, the method of laser acceleration in a plasma is now being actively developed. The results achieved in the acceleration method based on relativistic electron bunch–driven wake plasma waves are not as significant: the accelerating fields are about 50 kV/cm, the bunch charge being about 4 nC. Recent progress in producing short dense electron bunches raises the hope that very strong accelerating fields will also be achieved in PWFA research [13, 80].[1]

2. NEW POSSIBILITIES OF INCREASING THE ACCELERATING FIELD STRENGTH IN A PLASMA

In this section, we discuss new possibilities of further increasing the accelerating field. Recall that the maximum electric field of a relativistic space-charge wave in a plasma is $E_{\max} = \frac{\tilde{n}_p}{n_0}\sqrt{4\pi n_0 mc^2\gamma}$, where $\tilde{n}_p$ is the maximum density in the space-charge wave [24]. The ratio $\frac{\tilde{n}_p}{n_0}$ is governed by the way in which the space-charge wave is initiated. In experiments on laser excita-

[1] The reviews and descriptions of the theoretical and experimental results on this subject can be found in, e.g., [55, 56].

1063-780X/02/2802-0125$22.00 © 2002 MAIK "Nauka/Interperiodica"

tion, this ratio is less than 15% (LWFA), and, in experiments on the generation of plasma waves by electron bunches, it is about 3% (PWFA). According to [15–19, 48], for LWFA, we have $\frac{\tilde{n}_p}{n_0} = \frac{a^2}{\sqrt{1+a^2}}$, where $a = \frac{eE\lambda}{2\pi mc^2}$, E is the electric field, and $\lambda = \frac{2\pi}{\omega}$ is the laser wavelength. For the excitation of wake plasma waves by electron bunches (PWFA), this ratio is known to be approximately equal to $\frac{\tilde{n}_p}{n_0} \sim \frac{n_b}{n_0}$ [13], where n_b is the beam density. Consequently, the maximum electric field in a plasma wave can be increased by increasing the laser field and/or laser wavelength as well as the density of the electron bunch exciting the plasma wave (or by searching for new ways of generating plasma waves). Comparing the ratios $\frac{\tilde{n}_p}{n_0}$ for LWFA and PWFA gives [9]

$$\left(\frac{eE\lambda}{2\pi mc^2}\right)^2 \approx \frac{n_b}{n_0}. \quad (1)$$

thereby determining the electron density in the bunch n_b that is required to excite a plasma wave with the same maximum electric field as that of a laser-driven plasma wave. This relationship implies that, in order to generate such a plasma wave in PWFA, it is necessary to make the ratio n_b/n_0 as large as possible. Note that the case $n_b/n_0 \sim 1$ is not considered here, because it goes beyond the applicability range of the above expression for $E_{\max}$ in LWFA, which was derived under the assumption $a \ll 1$.

2.1. Excitation of Wake Fields by Laser Pulses in a Solid-State Plasma

Another way of increasing $E_{\max}$ is to initiate waves in higher density plasmas, in particular, in a solid-state plasma. This possibility can be realized, in particular, in semiconductor plasmas. However, the plasma density in semiconductors ($n_0 \sim 10^{14}$–10^{18} cm^{-3}) is lower than the gas plasma density that has already been achieved in experiments on LWFA (~10^{19} cm^{-3}). Consequently, in developing plasma-based charged particle accelerators, it seems natural to turn to the plasma of metals, in which the density of free electrons is as high as 10^{22}–10^{23} cm^{-3}. Chen *et al.* [59, 60] proposed a very daring but somewhat exotic[2] idea of implementing LWFA, which, however, involves solving the following challenging problems:

(i) launching laser light into a metal,

(ii) exciting space-charge waves in a metal plasma by laser light,

(iii) weakening the effect of multiple scattering of a beam of accelerated particles by the plasma electrons that occur between the channeling planes, and

(iv) solving a very important problem of preventing the destruction of crystals affected by extremely powerful laser radiation via utilizing very short ($\sim\omega_{pe}^{-1}$) laser pulses.

According to the Chen and Nable's estimates [59], the energy density required to generate accelerating fields of about 100 GeV/cm should be as high as 3×10^7 J/cm^3.

Recall that the electric field of a space-charge wave is governed to a large extent by the way in which it is generated. The authors of [59, 60] proposed to excite a plasma wave by laser light via either the method used in LWFA or the method suggested by Katsouleas *et al.* [56], which involves the interaction between laser radiation and a plasma whose density is made periodically nonuniform in space by an acoustic wave or with the help of a diffraction grating. The latter method is based on the three-wave interaction involving a laser wave, an ion acoustic wave, and a Langmuir plasma wave. The wave interaction can give rise to a plasma wave with the frequency ω_{pe} and wavenumber k_p only under the following conditions: $\omega \approx \omega_{pe}$, $\omega_s \ll \omega_{pe}$, the wavenumber of a laser wave in a plasma is close to zero, and the wavenumber of an ion acoustic wave is equal to k_p. The phase velocity $v_{ph} = \omega_{pe}/k_p$ of the excited plasma wave is close to c.

Another method suggested by the authors of [60] is the generation of wake plasma waves by short laser pulses (as is done in the LWFA scheme), in which case the condition $\omega \gg \omega_{pe}$ should be satisfied. Since $\omega_{pe} \approx 10^{16}$ s^{-1} in both methods, it is necessary to develop and create extrapowerful UV lasers. It is suggested that a plasma density of about ~10^{23} cm^{-3} will be achieved by ionizing the atoms of a solid body by the same UV laser. In this way, however, the fact that laser light will be strongly damped because of the strong absorption should also be taken into account. To answer the question of whether the very daring and interesting ideas expounded in [59, 60] can be implemented in practice, it is necessary to investigate the issue of how deep intense UV laser light can penetrate into a metal with allowance for the losses from ionization and photoeffect. Keeping in mind the fact that even very short laser pulses of enormous power will destroy a solid body, the authors of those two papers proposed to accelerate charged particles in optical fibers or thin metal films, in which case laser radiation may become easier to launch into a crystal. They are justified in suggesting that pre-

[2] Note that our views regarding "exotic" things may soon undergo a radical revision. The methods of collective acceleration (in particular, the scheme for accelerating charged particles by charge-density waves in a plasma), which were proposed in 1956, also seemed to be very exotic at that time.

liminary experiments in this direction can be performed with semiconductors in which the electron density is as high as ~10^{18} cm^{-3}. In this connection, we must point out the very interesting work by Kitson *et al.* [64], in which the phenomenon of anomalous penetration of visible laser pulses into a semiconductor was predicted theoretically and discovered experimentally. In the opinion of the authors of [66, 67], this anomaly can be attributed to the excitation of surface plasma waves.

Note that the maximum possible electric field strength in a steady-state space-charge wave in a plasma is limited by the condition that the velocity v_e acquired by the plasma electrons in the field of this wave is equal to the wave phase velocity v_{ph}, so that we have

$$E_{\max} = \sqrt{4\pi n_0 mc^2(2\gamma - 1)}. \quad (2)$$

Another idea (of no less importance) presented in [59, 60] is that of utilizing not only solid-state crystal bodies in order to raise the electric fields of laser-driven plasma waves but also of using the crystalline properties of solid bodies in order to generate high-brightness beams of accelerated particles through the channeling effect. In fact, under the channeling conditions, strong accelerating fields and the very high rates at which accelerated particles acquire energy can substantially lower the emittance of a beam of accelerated particles; however, it is necessary to take into account the fact that the channeling angle is approximately equal to $\psi \approx \sqrt{eU_b/\varepsilon_p}$, where eU_b is the depth of the potential well (or the height of the potential barrier that forms between the lattice planes of the crystal). For very high energies ε_p, this angle is very small. That is why the possibility of substantially reducing the emittance of a beam of charged particles accelerated to extremely high energies in crystals requires more detailed theoretical and experimental investigations. It may be that the ideas developed by the authors of [59, 60] will not be implemented in full measure in the near future. However, some aspects of the acceleration methods proposed in those papers can be used to create very-high-energy (about 10^{13}–10^{18} eV) particle accelerators.

A different scheme for charged-particle acceleration in crystals was proposed by Tajima *et al.* [62, 63] and was further developed in subsequent works. This scheme is based on the analogy with particle acceleration in microwave waveguides with periodically spaced metal or dielectric disks and implies acceleration via hard X radiation, for which a periodic crystalline structure plays the same role that periodically spaced disks play for microwave radiation in waveguides. The use of crystals for particle acceleration via hard X radiation on the basis of the Borman effect [62] eliminates the problems of launching laser radiation into a crystal and of guiding laser pulses over relatively large distances in a crystal. Tajima and Covenago [62] proposed to channel accelerated charged particles in a crystal in order to reduce their scattering. They also noted that Hofstadter had already originated analogous ideas in his unpublished paper. At this point, we should say a few words about the history of research on charged-particle acceleration in solids. Grishaev and Nasonov [61] suggested to accelerate charged particles by longitudinal polarization waves of an optically active matter that are driven by the beating of two electromagnetic waves and noted that the channeling effect can serve to reduce the divergence of a beam of accelerated particles due to their multiple scattering. Tajima and Covenago [62] proposed to accelerate charged particles in crystals by hard X radiation and to lower the divergence of a beam of accelerated particles by channeling them. They also studied some other aspects of this acceleration method. Examining the prospects for the new concept of accelerating charged particles by laser radiation in a solid body, Tajima and Covenago determined the electric field of a plasma wave in a metal from the relationship that was obtained for the maximum field of a nonlinear wave propagating in a plasma by solving the problem of natural waves. However, it is clear that the electric field should be estimated by solving the problem of induced oscillations and waves. Since the electric field of the plasma wave is very sensitive to the way in which the wave is excited, we think that deriving the final expression for the electric field requires solving the problem of the excitation of the plasma wave in the case at hand. In the wake-field acceleration scheme proposed in the rather interesting paper by Rosenzweig *et al.* [14], a high-energy electron beam is used to excite extremely nonlinear plasma oscillations in which acceleration occurs preferentially in the transverse direction.

Balakirev *et al.* [42] proposed a method for substantially increasing relativistic electron bunch–driven wake fields owing to the self-modulation of a long pulsed electron bunch in a plasma. The field excited by the bunch front affects the motion of the bunch electrons in such a way as to modulate the bunch density, or, in other words, to break the bunch into microbunches. Since the bunch is modulated at the plasma frequency, the wake fields generated by microbunches are coherent, so that the amplitude of the excited wake fields can increase substantially. However, the results of 2.5-dimensional simulations carried out by Batishchev *et al.* [34] show that, because of the unsteady dynamics of the self-consistent fields of a relativistic electron bunch with dimensions comparable to the skin depth, the wake field amplitude along the train of microbunches increases more gradually than in the case of a “rigid” bunch.

2.2. PWFA Experiments

Experimental investigations on PWFA were begun at the Argonne National Laboratory (ANL) [13, 14, 79–82, 86] and then were continued at the University of Tokyo [87]. The first experiments at ANL were carried out with 24.1-MeV beams (the electric charge of the driving bunch was 4 nC, the bunch dimensions being about 1 mm) and 16.6-MeV beams (the accelerated and diagnosed bunches had the same dimensions). The bunches were injected into a plasma whose density was varied from 10^{11} to 10^{13} cm^{-3}. The amplitude of the excited wake fields was measured to be about 6 MeV/m, which could only be explained by invoking the plasma and bunch nonlinearities. The corresponding simulations performed at this time were all one-dimensional (see, e.g., [13, 14]), and only some estimates were obtained with allowance for two- and three-dimensional effects. It was pointed out that three-dimensional effects should be systematically taken into account in investigating the role of the nonlinear behavior of the plasma and bunch particles. At the University of Tokyo, PWFA experiments were carried out with trains of six 500-MeV bunches. The plasma density was varied approximately within the same range as in the experiments at ANL. It was found that the amplitude of the excited wake field depends linearly on the plasma density in the range from 10^{11} to 10^{12} cm^{-3} [87]. The insufficiently high excitation rates of the wake fields and large radial displacements of the succeeding bunches in the train were not adequately explained at the theoretical level (these questions were mostly answered in our investigations). Further PWFA research is aimed at developing wake-field acceleration schemes for future linear colliders. At present, the PWFA scheme is being investigated most actively at ANL, the University of California at Los Angeles (UCLA), and the University of Southern California (USC) in collaboration with the Stanford Linear Accelerator Center (SLAC) on the 30-GeV linear accelerator at Stanford. Preliminary results from these very important experiments were reported at the annual meeting of the American Physical Society in October 2000. However, since these promising results were presented as abstracts of papers and have not yet been published in scientific journals, we cannot discuss them here. The photocathode in an accelerator created at ANL [79–82] is capable of ensuring the following bunch parameters: an electron energy of 200 MeV, a bunch charge of 100 nC, and a bunch duration of 20 ps. The photocathode is struck by light from a laser with a wavelength of 248 nm, a pulse duration of 2 ns, and an input energy of 8 MJ. This project is aimed at achieving acceleration rates of about 100 MeV/m.

2.3. Excitation of Wake Fields by Laser Pulses

Another promising method for exciting wake fields in a plasma makes use of short (picosecond and femtosecond) laser pulses with intensities of 10^{16} to 10^{19} W/cm^2. The results of recent PBWA experiments at UCLA are presented in [88–90] (recall that the acceleration scheme based on the beating between electromagnetic beam pulses was first proposed by Litvak [91]). In those experiments, relativistic plasma waves were resonantly excited in a plasma by the beating between two collinear beams from CO_2 lasers operating simultaneously at two different wavelengths. The acceleration of test electrons in a plasma was investigated using electron beams with a peak current of 200 mA, an electron energy of 2 MeV, and a pulse duration of 1 ns in order to measure the longitudinal wake fields, which themselves had a large relativistic factor equal to 34 (the relativistic factor of the excited wake field is large because of the large ratio of the laser frequency to the electron plasma frequency, $\gamma = \omega/\omega_{pe}$). The energy spectrum of the accelerated electrons was recorded using a special-purpose multisector magnet and a surface-barrier detector. The electrons were accelerated to an energy of 20 MeV over a distance of 1 cm, the acceleration rate being higher than 1.8 GeV/m. Clayton *et al.* [90] experimentally demonstrated the acceleration of electrons by relativistic plasma waves generated by stimulated Raman forward scattering of a short single-frequency laser pulse with a wavelength of 1.053 μm, a duration of 6×10^{-13} s, and a peak power of 8×10^{17} W/cm^2. The density of the plasma created by an auxiliary laser was varied in the range (1–2.5) $\times 10^{15}$ cm^{-3}, and the plasma itself was homogeneous over a length of 0.8 mm. Electron acceleration was observed to correlate with the generation of the first anti-Stokes satellite in the radiation spectrum. The calculations carried out in [90] showed that, by increasing the interaction length to 1.3 mm, it is possible to achieve acceleration rates as high as 1 GeV/cm. Beginning with the pioneering paper [4] by Tajima and Dawson, the PBWA method has been actively investigated not only theoretically and experimentally but also numerically [21, 88]. These investigations showed that the PBWA method works most efficiently with the theoretically predicted lengths of the laser pulses [92] because of the detuning of the plasma frequency from the difference between the laser frequencies. Numerical simulations showed that short acceleration lengths stem not from the plasma inhomogeneity but rather from the effect of the diffractive spreading of laser pulses on the Rayleigh length, in which case only about 2% of the injected particles are accelerated. The progress achieved in theoretical and numerical studies on plasma-based laser accelerators was reported in [57, 88], where the possibility of guiding laser pulses in plasma channels was also discussed in the context of preventing their radial spreading. The question of whether it is expedient to use plasma channels has already been discussed by Tajima and Dawson [4]. At present, it is proposed to use channels preformed in a special manner rather than rectangular channels. Since the most serious obstacle to acceleration is the diffractive spreading of the pulse, it is by the appropriate choice of the channel parameters that the acceleration length was significantly increased to

$$l_{acc} = \lambda_{pe}\frac{\omega^2}{\omega_{pe}^2}\left[1+\left(\frac{c}{V_{osc}}\right)^2\right], \qquad (3)$$

where $V_{\rm osc} = \dfrac{eE}{m\omega}$ is the electron oscillatory velocity in the laser field. In this case, the growth rate of the Raman instability is equal to

$$\gamma_{RS} = 2^{-3/2}\frac{V_{\rm osc}}{c}\frac{\omega_{pe}^2}{\omega}\sqrt{1-\left(\frac{4p\lambda_{pe}}{2\pi}\right)^2}, \qquad (4)$$

where $p^2 + h^2 = \left(\dfrac{2\pi}{\lambda_{pe}}\right)^2$, $p = h\tanh a_0$, and a_0 is the characteristic channel radius.

The guiding of intense laser pulses by plasma channels preformed in a special way was investigated in [57, 93]. It was shown that, in an empty channel, the growth rate of the Raman forward scattering instability decreases if $\pi a_0 < 0.13\lambda_{pe}$.

2.4. SMWFA Method

This method for charged particle acceleration has already been discussed above. Here, we analyze recent results on this subject. The SMWFA scheme is of considerable physical interest because the self-modulation of a laser pulse and the self-modulation of a relativistic electron bunch have much in common. As will be seen below, the self-modulation of a relativistic electron bunch is one of the most promising methods for exciting intense accelerating fields in a plasma. Moore *et al.* [94] carried out a detailed experimental investigation of the self-modulation of intense laser pulses with a power of 2 TW, a duration of 400 fs, an energy of 1 J, a wavelength of 1.054 nm, and an intensity of 5×10^{18} W/cm^2, the radius of the initial focal spot being 6 μm. In a plasma with a density of 1.4×10^{19} cm^{-3}, they succeeded in accelerating 10^8 plasma electrons to an energy of 30 MeV. The plasma wave was generated over a distance of 20 Rayleigh lengths. It was established that electron acceleration correlates with the generation of harmonics in the radiation spectrum. When the laser power was decreased by one-half or the plasma density was lowered, no high-energy electrons were observed. In the experiments of [95, 96], intense (10^{18} W/cm^2) laser pulses in a plasma were found to undergo self-channeling over a distance of 20 Rayleigh lengths. Papers [97, 98] were devoted to an experimental and theoretical study of the acceleration of the injected 3-MeV electrons by an intense wake wave, such that the accelerating electric field strength was 1.5–5 GV/m. The wake wave was excited by a 5-J laser pulse with a duration of 0.4–7 ps, which was guided in a preformed plasma channel with a density of 10^{19} cm^{-3}. It was shown that, in such plasma channels, long laser pulses (such that the laser power is low in comparison with the critical power for self-focusing) propagated a distance of three Rayleigh lengths. The self-modulation of a laser pulse led to the excitation of a plasma wave with a density variation of 6%. Since the amplitude of this plasma wave was far below the wavebreaking amplitude, the electrons could not be accelerated to megaelectronvolt energies. High-power laser pulses were observed to interact with a plasma in a strongly nonlinear fashion, giving rise to the filamentation instability. The pulses underwent self-focusing and self-modulation without generating high-energy resonant plasma electrons. The acceleration of a large number of plasma electrons to megaelectronvolt energies was observed with laser pulses whose power was 28 times higher than the relativistic self-focusing threshold. The radial structure of a wake wave excited during the self-modulation of a laser pulse in a homogeneous subcritical plasma was studied in detail by Andreev *et al.* [99]. The generation of an electron beam in the SMWFA scheme was thoroughly investigated by Chen *et al.* [100] both experimentally and by three-dimensional numerical simulations. They showed that the emittance of the generated 2-MeV electron beams is extremely small (0.06π mm mrad), in which case the main role is played not by the space-charge forces in the beam but by the nonlinearity of the plasma wave and the magnetic field of the wave (rather than the quasistatic magnetic field, as was asserted in many papers; see, e.g., [101, 102]). The interaction of short laser pulses with a transversely inhomogeneous plasma, relativistic filamentation, and field ionization were studied in [103–107]. The self-focusing of an intense ultrashort laser pulse and the accompanying processes (such as the generation and acceleration of ions, electron cavitation, the formation of channels, and the magnetic field generation) were investigated in detail in [101, 102, 108, 109].

2.5. PWFA Scheme with a Plasma Channel of Depressed Density

The PWFA research program proposed by Barow and Rosenzweig [81] relies on the use of plasma channels with a depressed electron density. The authors of [81] presented previous results obtained on the interaction of intense relativistic electron beams with plasmas in weak magnetic fields and suggestions for future work. This PWFA method deals with electron beams of density $n_b > n_p$. In such a situation, the plasma electrons escape from the propagating beam, thereby giving rise to an ion channel (the so-called ion focus regime). The PWFA experimental research program is being implemented in the accelerator at ANL. In recent papers [81, 110], it was proposed to use the so-called "blow-out" regime, which ensures that an electron beam propagates through a plasma with minimum distortions and losses. The model developed in those papers describes a nearly equilibrium electron beam propagating in such a manner through an underdense plasma, in particular, in the presence of an external magnetic field. The numerical results were obtained by solving the Vlasov–Maxwell equations for beam electrons and the hydrodynamic equations for plasma particles. A comparison between the results obtained from the equilibrium model and from the related simulations based on the macroparticle method showed that, under the conditions of collisional damping, the beam keeps its equilibrium. In addition, the possibility of using a relativistic electron bunch as an adiabatic lens was discussed. The blow-out regime, which has been recently proposed for the PWFA scheme [81, 110] and in which the plasma electrons are completely blown out from the beam region, has a number of advantages. When an intense electron beam with a sufficiently long duration propagates through a low-density ($n_0 < n_b$) plasma, the blow-out of all plasma electrons from the beam region gives rise to an ion channel. This regime was called the ion focus regime. The magnetic self-focusing forces cause the beam to propagate in the ion channel. The radius R_{eq} of the equilibrium beam propagating in a fully formed ion channel is equal to

$$R_{eq} = \frac{\varepsilon_n}{\sqrt{2\pi r_e n_0 \gamma_b}}, \qquad (5)$$

where ε_n is the normalized beam emittance and r_e is the classical radius of an electron. It is possible to single out three qualitatively different regions of the beam. The leading part of the beam (the beam head) is not focused by the plasma and therefore expands. The body of a beam propagating in the ion channel experiences the strongest focusing force. The transition region between the head of the beam and its body cannot be described in the linear-optics approximation because of the presence of plasma electrons. For a laser pulse of sufficiently short length L_0 ($2\pi L_0 \le \lambda_{pe}$), the evolution of both the beam head and the transition region has a substantial impact on the efficiency with which the beam is transported over long distances. The head of an ultrarelativistic beam expands freely in accordance with the value of the emittance, in which case the blow-out rate of the plasma electrons becomes slower. As a result, the succeeding parts of the beam experience a weaker focusing force, so that the beam is distorted to a significant extent. A simple one-dimensional model shows that, after a certain period of initial expansion, the beam is distorted at a very slow rate. It was also shown that the beam continues to expand until the plasma electrons are completely blown out from the beam region; in other words, the beam slowly evolves to a pinching regime and acquires an equilibrium structure. Barow *et al.* [110] proposed to use the blow-out regime in order to generate accelerating fields of 80–150 MeV/m by electron bunches with a charge of 90 nC. Below, we will describe the results from investigations of the formation of an ion channel due to the ion motion in self-consistent electromagnetic fields excited by a relativistic electron bunch.

3. 2.5-DIMENSIONAL NUMERICAL SIMULATION OF THE EXCITATION OF WAKE FIELDS BY A TRAIN OF RELATIVISTIC ELECTRON BUNCHES IN LOW- AND HIGH-DENSITY PLASMAS

The excitation of a steady-state nonlinear wake wave by a periodic train of relativistic electron bunches in a plasma was studied by Amatuni *et al.* [12], who showed that, when the plasma and bunch densities are comparable, the wave electric field increases with the relativistic factor of the train. Investigations of the nonlinear regime in PWFA experiments made clear the importance of three-dimensional effects [13, 14]. There are two different mechanisms that can produce strong wake fields and thus can be exploited in the physics of plasma-based accelerators. On the one hand, a short wide first bunch can excite large-amplitude wake waves capable of accelerating the succeeding bunches. On the other hand, a long narrow electron bunch can be well focused by its own magnetic field, provided that its space charge is completely neutralized by the plasma. The wake-field excitation is studied with COMPASS—a modified two-coordinate three-velocity axisymmetric, truly relativistic electromagnetic code [30–32]. Previously, this code was used to model inductive plasma accelerators [32], to simulate the interaction of relativistic beams with plasmas [33], and to investigate the propagation of a solitary relativistic electron bunch or a train of such bunches through both high-density and low-density plasmas [34]. Note that, in the experiments of [13, 14], the density of relativistic electron bunches was $n_b \le n_0/2$, in which case the transverse and longitudinal dimensions of the bunch, R_0 and L_0, were small in comparison with the skin depth c/ω_{pe}. Computer simulations [34, 35] showed that the transverse dimension of the bunch may change substantially as it propagates through the plasma. This results in significant changes in both the bunch density (by more than one order of magnitude) and the excited wake fields. It was shown that the amplitudes of the transverse and longitudinal fields increase as each next relativistic electron bunch is injected, but, unlike in the case of rigid bunches, their increase is not proportional to the number of injected bunches. A new electron accelerator created at the Kharkov Institute of Physics and Technology [69] is expected to be used in future experimental research on the acceleration of charged particles in intense wake fields. The operating parameters of the accelerator are as follows: the electron energy is W = 18–20 MeV, the number of electrons in a bunch is $N \approx 10^{10}$, the number of bunches is up to 20, and the modulation frequency of the bunches is 2797.16 MHz. It is proposed that, in experiments, relativistic electron bunches with dimensions comparable to the skin depth will be injected into a plasma whose density will be varied over a very wide range (by more than four orders of magnitude). The plasma is to be homogeneous to within several percent, so that the homogeneous plasma approximation is quite adequate for describing the experiment.

Since it is anticipated that no instabilities giving rise to the azimuthal plasma inhomogeneity would occur under the planned experimental conditions, a theoretical description can be based on the azimuthally symmetric mathematical model involving the injection of particles into the computation region and their escape from it. Under the conditions that are expected to prevail in experiments with the new accelerator, relativistic electron bunches will be injected into a plasma column with the length L = 100 cm, the radius R = 10 cm, the density in the range $n_0 = 10^{10}$–10^{14} cm^{-3}, and a minimum longitudinal density gradient. The numerical results presented below were obtained precisely for this experimental situation.

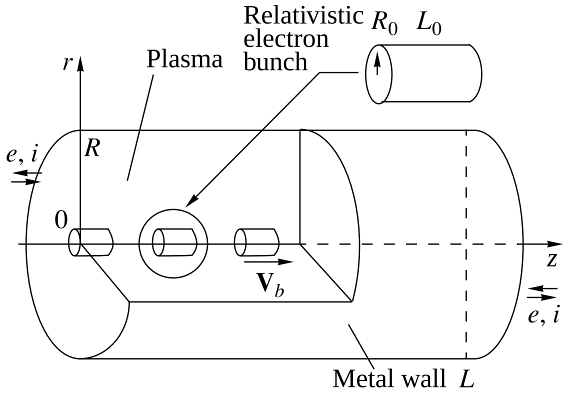

Fig. 1. Schematic of the model plasma–bunch system.

3.1. Mathematical Model

We describe the dynamics of a relativistic electron bunch by the relativistic Vlasov equations (the Belyaev–Budker equations) for the distribution functions of each of the plasma components and by the set of Maxwell's equations for the self-consistent electric (**E**) and magnetic (**B**) fields. The plasma–bunch system to be modeled is shown schematically in Fig. 1. At the initial instant, a two-component (m_i/m_e = 1840, where m_i and m_e are the ion and electron masses) cold plasma fills the entire computation region [0, L] × [0, R]. In simulations, we usually set L and R equal to 100 and 10, respectively. A finite number of relativistic electron bunches with the electron density

$$n(r, z) = n_b\theta(R_0 - r)\theta(V_b t - z + (n-1)\lambda_p)$$
$$\times\theta(z - V_p t + L_0 + (n-1)\lambda_p),$$

are injected into a plasma through the z = 0 plane. Here, $\theta(z)$ is the Heaviside step function; n is the order number of the injected bunch; $V_b = c\sqrt{1-1/\gamma_b^2}$ is the bunch velocity; the initial bunch dimensions L_0 and R_0 are equal to 0.4 and 0.5 cm, respectively; $\lambda_p = 2\pi c/\omega_{pe}$; and n_b is the mean bunch density. The bunch electrons and plasma particles can escape from the computation region through the boundary surfaces z = 0 and $z = L$. The plasma particles can also enter the computation region. The boundary conditions at the inner surface of the computation region assume a metal surface $r = R$ and a free escape of electromagnetic waves through the front and back surfaces. In our calculations, we use an explicit difference scheme. The excitation of wake fields by a train of bunches in a plasma was investigated in four series of simulations aimed at analyzing the dependences of the excited field on the number N_b of bunches injected into the plasma, on the bunch-to-plasma density ratio, on the repetition rate of the bunches, and on the ratio of the bunch radius R_0 to the skin depth c/ω_{pe}. The parameters of these four series are presented in the table.

The mass of the model particle is a function of its radial positions. In a lesser perturbed region (which is farther from the symmetry axis), the plasma is modeled by a relatively small number of particles. The total number of macroparticles is about 10^6. Note that all simulations were carried out with an advanced particle-in-cell (PIC) method implemented on a Pentium-133 PC.

3.2. Numerical Results and Discussion

Computer modeling showed that, as a relativistic electron bunch with $R_0 < c/\omega_{pe}$ and $L_0 < c/\omega_{pe}$ propagates in a plasma, its radius changes substantially. In contrast to the frequently used conditions $L_0 \gg c/\omega_{pe} > R_0$ or $R_0 \gg c/\omega_{pe} > L_0$, we simulated bunches with the initial dimensions $L_0 \approx R_0 < c/\omega_{pe}$ or $L_0 \approx R_0 \approx c/\omega_{pe}$, which correspond to those in the experiments of [13, 14]. In this case, the plasma and bunches are essentially non-

Parameters of the plasma and the bunches

Calculation version	Bunch density n_b, cm^{-3}	Plasma density n_0, cm^{-3}	Plasma frequency ω_{pe}, s^{-1}	Skin depth c/ω_{pe}, cm	Number of electrons in a bunch N
1	2×10^{10}	4×10^{10}	1.13×10^{10}	2.66	6.28×10^9
2	2×10^{10}	4×10^{11}	3.57×10^{10}	0.84	6.28×10^9
3	4.86×10^{10}	9.72×10^{10}	1.76×10^{10}	1.71	1.53×10^{10}
4	4.86×10^{10}	8.75×10^{11}	5.27×10^{10}	0.57	1.53×10^{10}

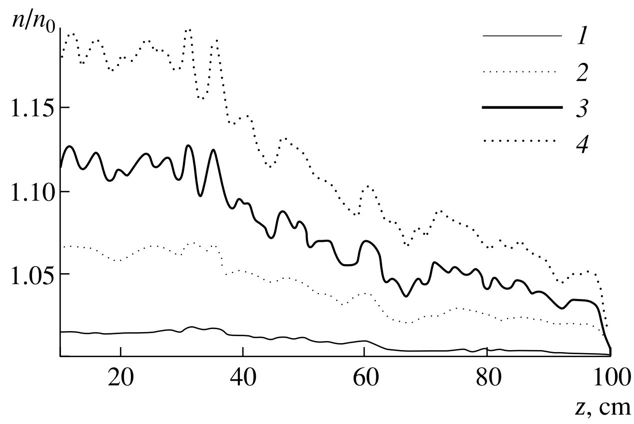

Fig. 2. Longitudinal profiles of the ion density obtained in calculation version 1 at the radius $r = 0.5$ cm at the times $t\omega_{pe}$ = (*1*) 70, (*2*) 120, (*3*) 150, and (*4*) 180.

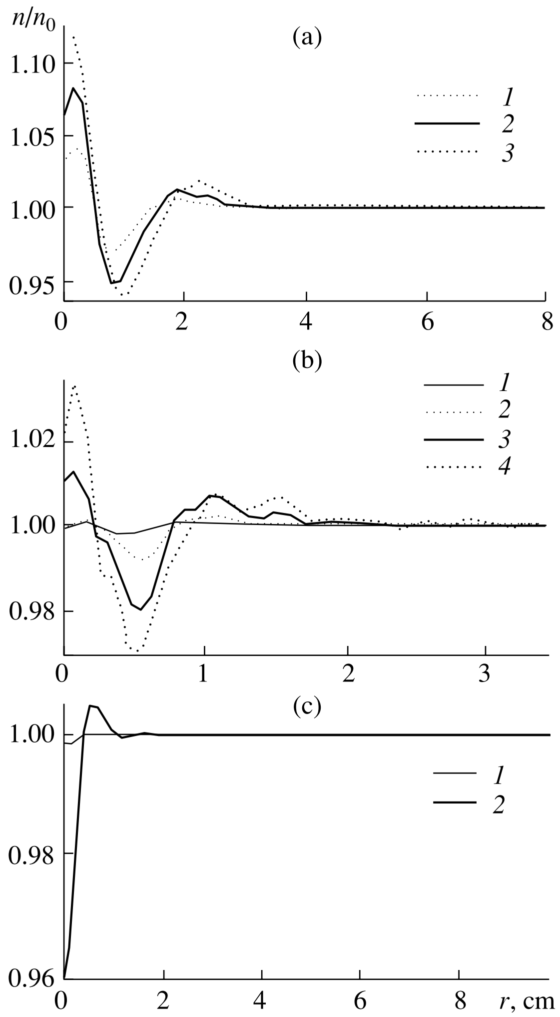

Fig. 3. Radial profiles of the ion density obtained (a) in calculation version 1 at the times $t\omega_{pe}$ = (*1*) 40, (*2*) 70, (*3*) 120, and (*4*) 180; (b) in calculation version 4 at the times $t\omega_{pe}$ = (*1*) 100, (*2*) 200, (*3*) 260, and (*4*) 300; and (c) in calculation version 3 at the times $t\omega_{pe}$ = (*1*) 100 and (*2*) 300.

linear. The numerical results obtained in [36] showed that the propagation of a relativistic electron bunch in a plasma is greatly affected by the ion motion. The time evolution of the longitudinal profile of the ion density n_i is illustrated in Fig. 2, which was obtained in calculation version 1 (see table). The time evolutions of the dependence of the ion density on the radial coordinate r are illustrated in Figs. 3a and 3b, which were obtained in calculation versions 4 and 3, respectively. From Figs. 2 and 3, we can see that the formation of an ion channel stems from the radial ion motion in self-consistent fields. At the axis of the system, the ion density is elevated and increases in the direction opposite to the propagation direction of the bunches. The elevated central density is higher than the unperturbed ion density by more than 15%. The characteristic time scales on which the ion channel forms are about a hundred inverse electron Langmuir frequencies; in other words, as may be expected from illustrative physical considerations, these scales are governed by the inverse ion Langmuir frequencies. The channel parameters are determined by the plasma-to-bunch density ratio and the ratio of the initial bunch radius R_0 to the skin depth c/ω_{pe}.

The amplitude of the ion density oscillations is seen to be substantially smaller than the averaged density. For comparison with the longitudinal profile of the ion density, Fig. 4 shows the longitudinal profile of the electron density. The electron density is seen to undergo only strong oscillations driven by the excited wake wave, and its averaged value remains essentially unchanged. In wake fields, the electrons oscillate without any significant change in their equilibrium positions. As a result, the electric field of the unneutralized positive ion charge accumulating at the system axis stabilizes the propagating electron bunches.

Figures 5 and 6 show longitudinal profiles of the longitudinal (E_z) and radial (E_r) electric fields at two times. One can see that the amplitudes E_r and E_z increase as each next bunch is injected, but, unlike in the case of rigid bunches, their increase is not proportional to the number of injected bunches. This dependence stems from the fact that, because of the charge and current neutralization processes, the bunch electrons undergo transverse oscillations in self-consistent fields. We can see that, owing to neutralization of the bunch space charge by a dense plasma, the azimuthal magnetic field is not fully canceled and thus drives the bunch electrons into radial motion, leading to a significant distortion of the bunch shape and a charge density redistribution within the bunches. As a result, the wake fields are excited in an unsteady fashion, which is unfavorable for charged particle acceleration. The shape of the envelope of the bunches is seen to deviate substantially from the Bennett equilibrium shape. The electromagnetic fields generated by the bunches propagating in a plasma redistribute the ion plasma density and give rise to the averaged electric field, which promotes the focusing of the bunch electrons. Hence, the radial expansion of the bunches is prevented to a large extent by the formation of the plasma channel due to the radial motion of the plasma ions. A train of bunches propagating in a stable and steady-state fashion in the fully developed ion channel excites steady-state wake fields suitable for accelerating the injected electrons. We can thus conclude that the nonlinear processes analyzed in this section have a beneficial effect on the bunch propagation and, accordingly, on the excitation of the accelerating fields by the bunches. The investigation of the three-dimensional nonlinear behavior of the beam–plasma system was undertaken in order to provide a better insight into the fundamental physics governing the acceleration and focusing of charged particles by the wake waves. Numerical experiments show that the ion motion in the self-consistent fields excited by a train of relativistic electron bunches gives rise to an ion channel at the symmetry axis of the system; in turn, the ion channel makes the bunch propagation more stable, so that the accelerating fields excited by the bunches are stronger.

4. 2.5-DIMENSIONAL NUMERICAL SIMULATION OF THE EXCITATION OF WAKE FIELDS DURING THE SELF-MODULATION OF A LONG RELATIVISTIC ELECTRON BUNCH

In this section, we describe the results of numerical modeling of the wake-field excitation by a relativistic electron bunch in a dense plasma. The results presented were obtained using a two-coordinate three-velocity model in which the complete set of equations for the bunch–plasma interaction consists of the relativistic Vlasov equations for the bunch electrons, the nonlinear Vlasov equations for each of the plasma components, and nonlinear Maxwell's equations for the self-consistent electromagnetic fields. Our computer modeling showed that the nonlinear dynamics of the plasma and bunch particles leads to a significant self-modulation of the density of a long bunch, in which case the amplitude of the excited wake fields substantially increases.

4.1. 2.5-Dimensional Numerical Modeling of the Wake-Field Excitation by a Long Relativistic Electron Bunch

The physics of plasma accelerators usually deals with two regimes of the bunch–plasma interaction accompanied by the generation of large-amplitude wake plasma waves. In the first regime, a large-amplitude wake field excited by a short wide first bunch can accelerate the succeeding bunches in the train. In the second regime, a long narrow relativistic electron bunch can be strongly focused by its own magnetic field, provided that its space charge is completely neutralized by the plasma. In a wake electric field, the bunch electrons experience not only transverse forces but also strong longitudinal forces. Longitudinal wake

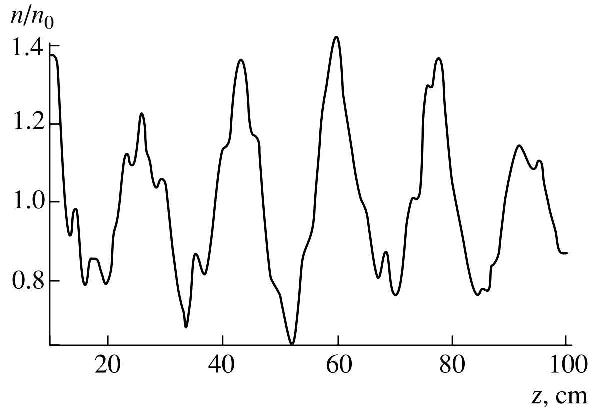

Fig. 4. Longitudinal profile of the electron density obtained in calculation version 1 at the radius $r = 0.5$ cm at the time $t\omega_{pe} = 180$.

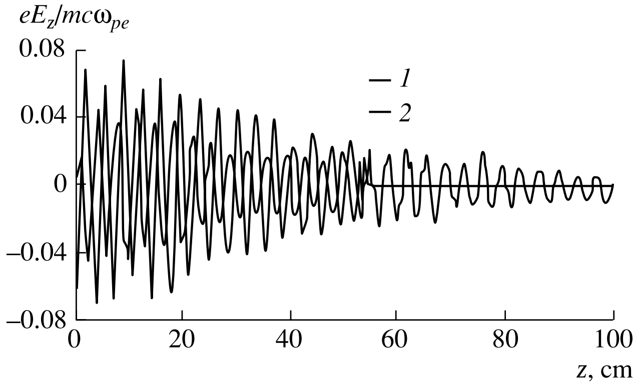

Fig. 5. Profiles of the longitudinal electric field E_z along the z-axis obtained in calculation version 4 at the radius $r = R_0 = 0.5$ cm at the times (*1*) $t = 100\omega_{pe}^{-1}$ and (*2*) $t = 200\omega_{pe}^{-1}$.

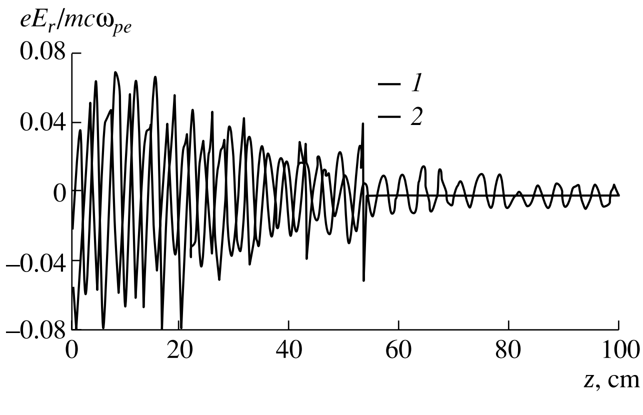

Fig. 6. Profiles of the radial electric field E_r along the z-axis obtained in calculation version 4 at the radius $r = R_0 = 0.5$ cm at the times (*1*) $t = 100\omega_{pe}^{-1}$ and (*2*) $t = 260\omega_{pe}^{-1}$.

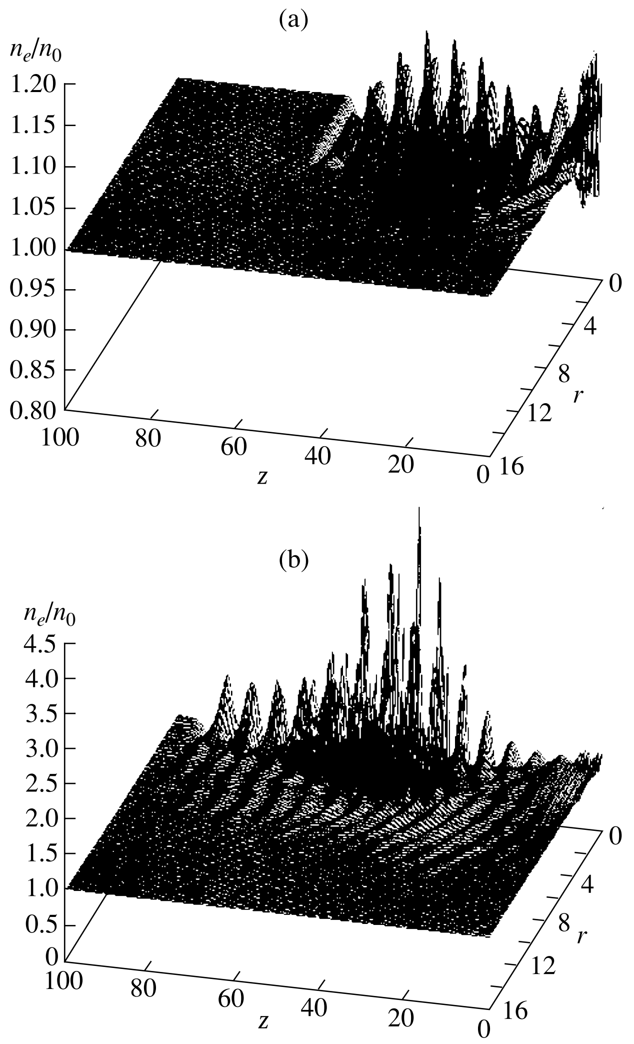

Fig. 7. Spatial distributions of the electron plasma density at the times (a) $t = 60\omega_{pe}^{-1}$ and (b) $t = 60\omega_{pe}^{-1}$. The coordinates are normalized to the skin depth c/ω_{pe}.

fields give rise to a longitudinal modulation (with the period $\lambda_p = 2\pi c/\omega_{pe} = 3.36 \times 10^6/\sqrt{n_0}$ cm) of the originally uniform electron bunch, thereby breaking it into microbunches. In particular, in a plasma with a particle density of 10^{16} cm^{-3}, the modulation period is 0.3 mm. The effect of the longitudinal modulation of relativistic electron bunches by the wake fields can be used to develop plasma modulators of high-density electron beams. Now, we should say a few words about another aspect of the modulation phenomenon. Since the modulation frequency coincides with the plasma frequency, the wake fields of the microbunches are coherent. Consequently, the modulation of an electron bunch leads to an increase in the wake-field amplitude behind the bunch. This effect makes it possible to use long relativistic electron bunches to generate intense wake fields in a plasma. It is important to note that long laser pulses can also undergo longitudinal modulation at the plasma frequency [28]. The modulation of long electron bunches by longitudinal wake fields in a plasma was investigated theoretically by Balakirev *et al.* [42]. The results of one-dimensional numerical modeling of the nonlinear dynamics of the bunch modulation showed that the modulation of a long electron bunch propagating in a plasma increases the amplitude of the excited wake wave. This effect is explained as being due to the coherence of the wake fields generated by microbunches resulting from the modulation of a long bunch at the plasma frequency. The one-dimensional approximation applies only to relativistic electron bunches with a sufficiently large radius ($2\pi R_0/\lambda_p \gg 1$).

The results presented in this section were obtained from 2.5-dimensional numerical modeling of the wakefield excitation by long relativistic electron bunches [38–41] with the COMPASS code [30–32].

4.2. Mathematical Model and Parameter Choice

The dynamics of a relativistic electron bunch is described by the relativistic Vlasov equations (the Belyaev–Budker equations)

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{V}_\alpha \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} + eZ_\alpha\left(\mathbf{E} + \frac{1}{c}\mathbf{V}_\alpha \times \mathbf{B}\right) \cdot \frac{\partial f_\alpha}{\partial \mathbf{p}} = 0$$

for the distribution functions $f_\alpha(\mathbf{r}, \mathbf{p})$ of each plasma component and by Maxwell's equations for the self-consistent electric and magnetic fields. At the initial instant, a two-component ($m_i/m_e = 1840$) cold plasma fills the entire computation region $[0, L] \times [0, R]$ with the length $L = 100$ cm and radius $R = 10$ cm. A cold relativistic electron bunch is injected into the plasma through the $z = 0$ plane. The bunch velocity is $V_b = c\sqrt{1 - 1/\gamma_b^2}$, and the initial bunch radius is $R_0 = 4c/\omega_{pe}$. The plasma and bunch particles can escape freely from the computation region through the two boundary surfaces $z = 0$ and $z = L$ and are elastically reflected from the $r = R$ surface. Cold plasma electrons and ions can also return to the computation region from the buffer zones $z < 0$ and $z > L$. The boundary conditions for the electromagnetic fields imply the existence of a metal wall at the cylindrical surface $r = R$ and free emission of electromagnetic waves from the right and left boundaries. In calculations, we used an explicit scheme. The mass of the model particle is a function of the radial coordinate, and the total particle number is about 10^6. Like the calculations described in Section 3, all simulations were carried out with a Pentium-133 PC using an advanced PIC algorithm. Our simulations showed that, for $\gamma_b = 5$, the bunch-to-plasma density ratio n_b/n_0 increases from the initial value 0.018 to 0.04 already at $t = 60\omega_{pe}^{-1}$. At the time $t = 100\omega_{pe}^{-1}$, the maximum electron density in the bunch becomes comparable to the

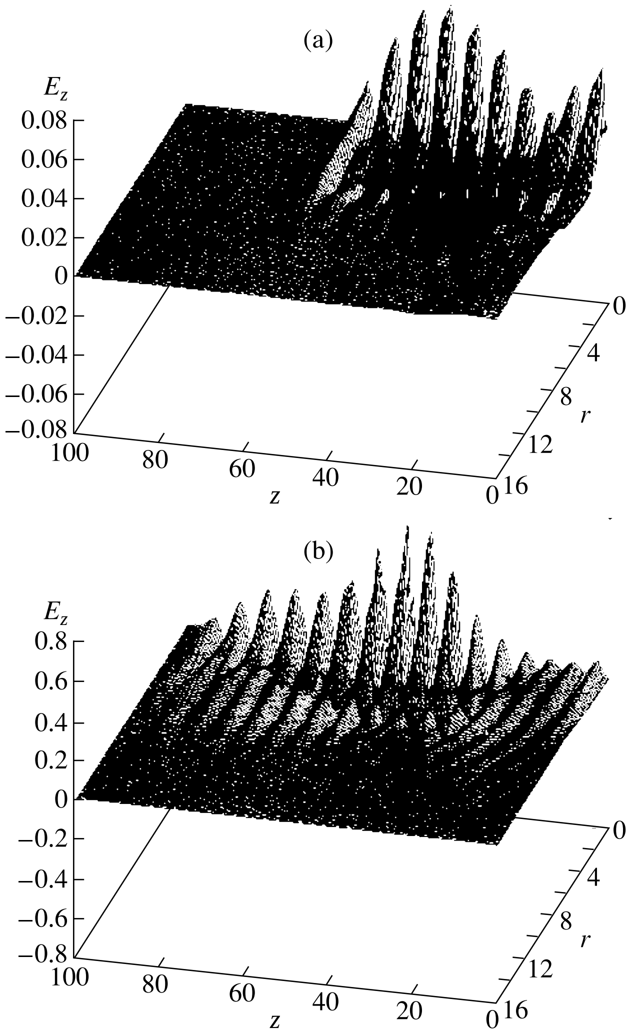

Fig. 8. Spatial distributions of the longitudinal electric field E_z (in units of $m_e\omega_{pe}c/e$) at the times (a) $t = 60\omega_{pe}^{-1}$ and (b) $t = 100\omega_{pe}^{-1}$. The coordinates are normalized to the skin depth c/ω_{pe}.

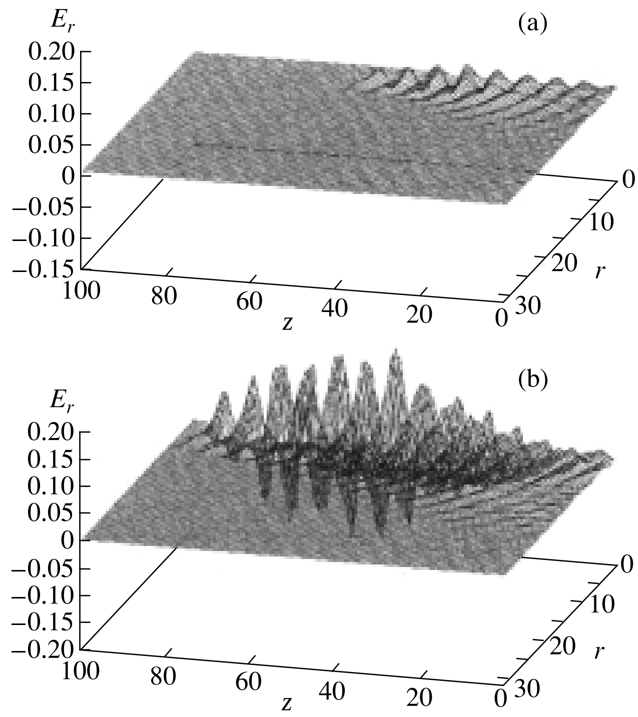

Fig. 9. Spatial distributions of the radial electric field E_r (in units of $m_e\omega_{pe}c/e$) at the times (a) $t = 60\omega_{pe}^{-1}$ and (b) $t = 100\omega_{pe}^{-1}$. The coordinates are normalized to the skin depth c/ω_{pe}.

plasma density, thereby indicating a very strong bunch modulation. In this case, the electron plasma density n_e also becomes modulated very strongly. The spatial distributions of n_e at the times $t = 60\omega_{pe}^{-1}$ and $t = 100\omega_{pe}^{-1}$ are shown in Figs. 7a and 7b, respectively. We can see that, at the time $t = 100\omega_{pe}^{-1}$, the maximum amplitude of n_e is larger than the initial amplitude by a factor of 4.5. The spatial distributions of the longitudinal (E_z) and radial (E_r) electric fields computed for the same times as in Fig. 7 are shown in Figs. 8 and 9, respectively.

4.3. Discussions and Conclusions

The numerical experiments described above show that the nonlinear dynamics of both the bunch electrons and the electron and ion plasma components lead to the following effects. The radii of the bunches vary over a very wide range. The self-modulation of a long bunch gives rise to a very strong modulation of the bunch and plasma densities, thereby substantially increasing the amplitude of the excited wake electric fields. However, it is necessary that the bunch length be optimum, because the self-modulation of longer bunches does not increase the amplitude of the excited electric fields. The results of numerical modeling show that even a very low-density relativistic electron bunch can perturb the plasma so strongly that the density perturbation amplitude will become comparable with the initial particle density of the plasma. This conclusion indicates that the plasma electrons cannot be described in the linear approximation. The results obtained show that both the effect of the self-modulation of long relativistic electron bunches and the use of a train of bunches hold promise for the generation of accelerating fields that would be far stronger than those achievable in conventional accelerators. Further investigations in this direction will provide the physical basis for the development and creation of a new generation of devices capable of accelerating charged particles at higher rates.

5. EXCITATION OF WAKE FIELDS BY A RELATIVISTIC ELECTRON BUNCH IN A MAGNETIZED PLASMA

Another promising way of accelerating charged particles is to excite wake fields by a relativistic electron bunch in a magnetized plasma [25–27]. Here, we present the main results obtained on the excitation of wake fields by an individual bunch in a magnetized plasma. In our opinion, this scheme is the most advantageous: because of the nonresonant character of the wake field excitation, it is only slightly sensitive to longitudinal density variations peculiar to a real plasma. Additionally, in order to prevent the development of electromagnetic filamentation or the onset of slipping instabilities, as well as other kinds of instabilities (see, e.g., [28]), it is worthwhile to apply a stabilizing external longitudinal magnetic field. As will be seen below, the stabilizing field not only serves to suppress instabilities but also gives rise to a large number of new wave branches, thereby substantially expanding the possibilities of the wake field acceleration scheme. In this section, we determine the wake field generated by an axisymmetric relativistic electron bunch propagating along the z-axis in a magnetized plasma, assuming that the ions are immobile and neglecting the electron thermal motion. The expression for the longitudinal component of the electric field excited by an annular relativistic electron bunch in an unbounded magnetized plasma was obtained in [25]:

$$E_z = \frac{2Q_0\omega_{pe}^2}{V_b^2\gamma_b^2}\frac{\tau}{(\tau^2+\mu^2)^{3/2}}$$
$$\times(\sqrt{\mu^2+\tau^2}\sin\sqrt{\tau^2+\mu^2}+\cos\sqrt{\tau^2+\mu^2}),$$

where $\tau = \omega_{pe}\left(t - \frac{z}{V_b}\right)$, $\mu = \frac{\omega_{pe}R_0}{V_b\gamma_b}$, ω_{pe} is the Langmuir frequency of the plasma electrons; and V_b, R_0, and Q_0 are the velocity, radius, and electric charge of the bunch, respectively.

Far behind the bunch, the wake field falls off as $\tau^{-3/2}$, because the group velocity of plasma oscillations in a sufficiently strong magnetic field is finite. Because of the emission of plasma waves from the axial region, the wake field decreases in the longitudinal direction.

We consider a waveguide partially filled with a plasma, i.e., a waveguide with a vacuum gap between the plasma surface $r = a$ and the perfectly conducting wall $r = b$, and assume that the waveguide is placed in an external magnetic field.

The field distribution over the waveguide cross section is governed by the transverse wavenumbers. The ranges $\lambda_{1,2}^2 > 0$ and $\lambda_{1,2}^2 < 0$ correspond to the spatial and surface modes, respectively. The complex values of $\lambda_{1,2}^2$ refer to a hybrid mode. The boundaries of the region where $\lambda_{1,2}^2$ are complex are determined by the inequalities $\omega_1 > \omega > \omega_2$, where

$$\omega_{1,2} = kc\frac{2\omega_{pe}^2+\omega_{He}^2 \pm (\omega_{pe}^4+\omega_{He}^2\omega_{pe}^2-\omega_{He}^2k^2c^2)^{1/2}}{\omega_{He}^2+4k^2c^2},$$

ω_{He} is the gyrofrequency of the plasma electrons, and k is the longitudinal wavenumber.

In order for a relativistic electron bunch to excite a hybrid mode, the relativistic factor of the bunch should satisfy the condition $\gamma_b > \frac{\omega_{He}}{2\omega_{pe}}$.

The electromagnetic field distribution and the frequency of a hybrid mode that synchronously accompanies the bunch were obtained numerically for the following parameters of the plasma waveguide: $\frac{\omega_{He}}{\omega_{pe}} = 6.3$, $\frac{\omega_{pe}a}{c} = 23.3$, $\frac{b}{a} = 2.4$, and $\gamma_b = 4.6$, in which case the frequency of the wake hybrid mode is equal to $0.35\omega_{pe}$. It was found that, at the radius $\frac{r}{a} = 0.8$, the absolute value of the longitudinal component of the electric field has a pronounced maximum, which corresponds to an energy conversion factor equal to $R_E = \left|\frac{E_{z\max}}{E_z(r=0)}\right| = 37$.

Recall that the energy conversion factor is defined as the ratio of the amplitude of the electric field accelerating a driven bunch to the amplitude of the electric field decelerating a driving bunch (the bunch exciting the wake field). Such a large value of R_E indicates that the maximum energy the driven bunch can gain during acceleration is significantly higher (by a factor of R_E) than the initial energy of the driving bunch.

Hence, it is shown that, for a certain relation among the parameters of the plasma–bunch–magnetic field system, the hybrid nature of the wake waves, which are excited by a relativistic electron bunch in a magnetized plasma and are a superposition of the surface and spatial modes, makes it possible to accelerate the driven bunch to the maximum energy

$$\varepsilon_{\max} = mc^2(R_E\gamma_b - 1),$$

which is many times higher than the initial energy of the driving bunch (even when the bunch is initially unmodulated in the longitudinal direction).

6. CONCLUSION

We have reviewed the results from theoretical and experimental investigations as well as from mathematical modeling of the wake-field generation by both charged-particle bunches and laser radiation in a plasma and have analyzed the wake-field acceleration of charged particles. A new, substantially modified version of the PBWA scheme has recently been proposed—a version based on the self-modulation of a laser pulse. The most impressive results on plasma methods of charged particle acceleration were obtained in the LWFA experiments [19–22], in which the accelerating fields at short distances were as strong as (1.5–20) × 10^8 V/cm and the particles were accelerated to energies of 100–300 MeV over distances of about 1 cm. An interesting fact has been established: for a certain relation among the parameters of the plasma–bunch–magnetic field system, the hybrid nature of the wake waves (which are excited by a relativistic electron bunch in a magnetized plasma and are a superposition of the surface and spatial modes) makes it possible to accelerate the driven bunch to an energy ε_{max} that is many times higher than the initial energy of the driving bunch (even when the bunch is initially unmodulated in the longitudinal direction). We have discussed the formation of an ion channel as a result of the radial ion motion in self-consistent electromagnetic fields excited by a train of relativistic electron bunches. The parameters of the fully developed channel are determined by the plasma-to-bunch density ratio and the ratio of the bunch radius to the skin depth. The effective dimensions of the channel and its "depth" (i.e., the elevated ion density at the channel axis) increase monotonically both in time and in the direction opposite to the propagation direction of the bunches. The formed ion channel stabilizes the propagation of relativistic electron bunches, which thus generate stronger accelerating fields. The results of 2.5-dimensional numerical modeling of the wake-field excitation during the self-modulation of a long relativistic electron bunch showed that the maximum electron density in the bunch becomes comparable to the plasma density and the amplitude of the plasma density perturbations becomes larger than the initial plasma density by a factor of 4.5. This indicates a very strong modulation of both the bunch density and the plasma density. That is why, even in the above case of a low-density bunch (in which the unperturbed electron density is about two orders of magnitude lower than the plasma density), it is incorrect to describe the plasma in the linear approximation. The amplitude of the longitudinal field is about 0.8 of the maximum electric field that can be generated in the plasma, and the amplitude of the radial field is about 0.4 of the maximum possible field. An important point is that the field amplitude increases only over a short distance along a relativistic electron bunch; hence, it would be of no use to operate with bunches whose length exceeds the distance over which the longitudinal field amplitude is maximum, because doing so would provide no additional increase in the excited wake field. The results obtained with allowance for all possible nonlinearities give a better insight into the three-dimensional behavior of relativistic electron bunches in a plasma and may help to ensure the optimum conditions for the wake-field generation during the dynamic self-modulation of the bunches. The results of investigations of the excitation of accelerating fields by an individual relativistic electron bunch or by a train of such bunches in a plasma (in particular, in the presence of an external magnetic field) make it possible to evaluate the potentialities of the wake-field acceleration method and to analyze whether it can serve as a basis for creating a new generation of devices capable of accelerating charged particles at substantially higher (by two to three orders of magnitude) rates in comparison with those achievable in classical linear resonance accelerators.

REFERENCES

1. G. I. Budker, in *Proceedings of the CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, 1956*, Vol. 1, p. 68.
2. V. I. Veksler, in *Proceedings of the CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, 1956*, Vol. 1, p. 80.
3. Ya. B. Fainberg, in *Proceedings of the CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, 1956*, Vol. 1, p. 84.
4. T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
5. Ya. B. Fainberg, Usp. Fiz. Nauk **93**, 617 (1967) [Sov. Phys. Usp. **10**, 750 (1968)].
6. Ya. B. Fainberg, Fiz. Plazmy **13**, 607 (1987) [Sov. J. Plasma Phys. **13**, 350 (1987)].
7. Ya. B. Fainberg, Fiz. Plazmy **20**, 613 (1994) [Plasma Phys. Rep. **20**, 549 (1994)].
8. Ya. B. Fainberg, Fiz. Plazmy **23**, 275 (1997) [Plasma Phys. Rep. **23**, 251 (1997)].
9. Ya. B. Fainberg, Fiz. Plazmy **26**, 362 (2000) [Plasma Phys. Rep. **26**, 335 (2000)].
10. P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
11. T. Katsouleas, Phys. Rev. A **33**, 2056 (1986).
12. A. Ts. Amatuni, M. R. Magomedov, E. V. Sekhposyan, and S. S. Elbakyan, Fiz. Plazmy **5**, 85 (1979) [Sov. J. Plasma Phys. **5**, 49 (1979)].
13. J. B. Rosenzweig, Preprint No. 90/40 (FERMILAB, Batavia, 1990).
14. J. B. Rosenzweig, B. N. Breizman, T. Katsouleas, and J. J. Su, Phys. Rev. A **44**, R6189 (1991).
15. N. E. Andreev, L. M. Gorbunov, V. I. Kirsanov, *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **55**, 551 (1992) [JETP Lett. **65**, 571 (1992)].
16. J. Krall, A. Ting, E. Esarey, and P. Sprangle, Phys. Rev. E **48**, 2157 (1993).
17. T. M. Antonsen and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).
18. P. Sprangle, E. Esarey, J. Krall, and G. Joyce, Phys. Rev. Lett. **69**, 2200 (1992).
19. K. Nakajima, T. Kawakubo, H. Nakanishi, *et al.*, in *Proceedings of the Conference on Advanced Accelerator Concepts, Fontana, 1994*, p. 145 [AIP Conf. Proc. **335**, 145 (1995)].

20. A. Modena, Z. Najimidin, A. E. Dangor, *et al.*, Nature (London) **337**, 806 (1995).
21. D. Umstadter, J. K. Kim, and E. Dodd, Phys. Rev. Lett. **76**, 2073 (1996).
22. A. Ogata, H. Nakanishi, T. Kawakubo, *et al.*, in *Proceedings of the Conference on Advanced Accelerator Concepts, Fontana, 1994*, p. 501 [AIP Conf. Proc. **335**, 501 (1995)].
23. S. V. Bulanov and A. S. Sakharov, Fiz. Plazmy **26**, 1074 (2000) [Plasma Phys. Rep. **26**, 1005 (2000)].
24. A. I. Akhiezer and R. V. Polovin, Zh. Éksp. Teor. Fiz. **30**, 915 (1956) [Sov. Phys. JETP **3**, 696 (1956)].
25. V. A. Balakirev, V. I. Karas', I. V. Karas', and G. V. Sotnikov, Vopr. At. Nauki Tekh., Ser.: Plazm. Élektron. Novye Metody Uskoreniya **1** (2), 113 (2000).
26. V. A. Balakirev, V. I. Karas', I. V. Karas', and Ya. B. Fainberg, in *Proceedings of the XXVII Zvenigorod Conference on Plasma Physics and Controlled Nuclear Fusion, Zvenigorod, 2000*, p. 180.
27. V. A. Balakirev, I. V. Karas', and G. V. Sotnikov, Fiz. Plazmy **26**, 948 (2000) [Plasma Phys. Rep. **26**, 889 (2000)].
28. C. Joshi, T. Katsouleas, J. M. Dawson, *et al.*, IEEE J. Quantum Electron. **QE-23**, 1571 (1987).
29. V. A. Balakirev, G. V. Sotnikov, and Ya. B. Fainberg, Fiz. Plazmy **22**, 634 (1996) [Plasma Phys. Rep. **22**, 573 (1996)].
30. O. V. Batishev, I. V. Karas', O. N. Shulika, *et al.*, Bull. Am. Phys. Soc. **45** (7), 72 (2000).
31. O. V. Batishchev, V. I. Karas', V. D. Levchenko, *et al.*, Fiz. Plazmy **20**, 654 (1994) [Plasma Phys. Rep. **20**, 587 (1994)].
32. V. I. Karas', O. V. Batishchev, Yu. S. Sigov, and Ya. B. Fainberg, Part. Accel. **37–38**, 281 (1992).
33. O. V. Batishchev, V. B. Krasovitskiœ, Yu. S. Sigov, *et al.*, Fiz. Plazmy **19**, 738 (1993) [Plasma Phys. Rep. **19**, 379 (1993)].
34. O. V. Batishchev, V. I. Karas', Yu. S. Sigov, and Ya. B. Fainberg, Fiz. Plazmy **20**, 650 (1994) [Plasma Phys. Rep. **20**, 583 (1994)].
35. V. I. Karas', Ya. B. Fainberg, V. D. Levchenko, and Yu. S. Sigov, in *Proceedings of the Particle Accelerator Conference and International Conference on High-Energy Accelerators, Dallas, 1995*, Vol. 2, p. 1233.
36. V. I. Karas', I. V. Karas', V. D. Levchenko, *et al.*, Fiz. Plazmy **23**, 311 (1997) [Plasma Phys. Rep. **23**, 285 (1997)].
37. N. I. Aœzatskiœ, V. A. Balakirev, A. N. Dovbnya, *et al.*, in *Proceedings of the XXVII Zvenigorod Conference on Plasma Physics and Controlled Nuclear Fusion, Zvenigorod, 2000*, p. 156.
38. V. A. Balakirev, V. I. Karas', Ya. B. Fainberg, *et al.*, in *Proceedings of the 12th International Conference on High-Power Particle Beams, Haifa, 1998*, Vol. 2, p. 392.
39. V. I. Karas', V. A. Balakirev, Ya. B. Fainberg, *et al.*, in *Proceedings of XVIII International Symposium on Discharges and Electrical Insulation in Vacuum, Eindhoven, 1998*, Vol. 2, p. 781.
40. V. I. Karas', V. A. Balakirev, Ya. B. Fainberg, *et al.*, Vopr. At. Nauki Tekh., Ser.: Plazm. Élektron. Novye Metody Uskoreniya **1** (2), 122 (2000).
41. V. I. Karas', V. A. Balakirev, Ya. B. Fainberg, *et al.*, J. Tech. Phys. **41** (1), 293 (2000).
42. V. A. Balakirev, G. V. Sotnikov, and Ya. B. Fainberg, Fiz. Plazmy **22**, 165 (1996) [Plasma Phys. Rep. **22**, 151 (1996)].
43. R. Keinigs and M. E. Jones, Phys. Fluids **30**, 252 (1987).
44. A. Ts. Amatuni, E. V. Sekhposyan, A. G. Khachatryan, and S. S. Elbakyan, Fiz. Plazmy **21**, 1000 (1995) [Plasma Phys. Rep. **21**, 945 (1995)].
45. E. Esarey, J. Krall, and P. Sprangle, Phys. Rev. E **48**, 2157 (1993).
46. D. Umstadter, E. Esarey, and J. Kim, Phys. Rev. Lett. **69**, 2204 (1992).
47. P. Sprangle and E. Esarey, Phys. Rev. Lett. **67**, 2021 (1991).
48. V. A. Balakirev, A. O. Ostrovskiœ, and Yu. V. Tkach, Vopr. At. Nauki Tekh., Ser.: Tekh. Fiz. Éksp. **6**, 168 (1990).
49. A. Ting, C. I. Moore, K. Krushelnik, *et al.*, IEEE Trans. Plasma Sci. **PS-26**, 611 (1998).
50. K. Nakajma and A. Ogata, in *Proceedings of the 11th Symposium on Accelerator Science and Technology, Harima, 1997*, p. 36.
51. R. Assmann, P. Chen, F. Decker, *et al.*, in *Proceedings of the 1999 Particle Accelerator Conference, New York, 1999*, p. 330.
52. S. V. Bulanov, V. A. Vshivkov, G. I. Dudnikova, *et al.*, Fiz. Plazmy **23**, 284 (1997) [Plasma Phys. Rep. **23**, 259 (1997)].
53. N. E. Andreev, L. M. Gorbunov, and A. A. Frolov, Fiz. Plazmy **24**, 888 (1998) [Plasma Phys. Rep. **24**, 825 (1998)].
54. V. A. Balakirev, V. I. Karas', A. P. Tolstoluzhskiœ, and Ya. B. Fainberg, Fiz. Plazmy **23**, 316 (1997) [Plasma Phys. Rep. **23**, 290 (1997)].
55. J. Krall and G. Joyce, Phys. Plasmas **2**, 1326 (1995).
56. T. Katsouleas and W. Mori, in *Proceedings of the Conference on Advanced Accelerator Concepts, Fontana, 1994*, p. 112 [AIP Conf. Proc. **335**, 112 (1995)].
57. P. Sprangle, B. Hafizi, J. R. Penano, *et al.*, Phys. Rev. Lett. **85**, 5110 (2000).
58. L. M. Gorbunov and V. I. Kirsanov, Zh. Éksp. Teor. Fiz. **93**, 509 (1987) [Sov. Phys. JETP **66**, 290 (1987)].
59. P. Chen and R. Nable, in *Advanced Acceleration Concepts*, Ed. by S. Chattopadhyay, S. McCullough, and P. Dahl (American Inst. of Physics, New York, 1997), p. 273.
60. P. Chen, Z. Huang, and R. Ruth, in *Proceedings of the 4th Tamura Symposium on Accelerator Physics*, Ed. by T. Tajima (American Inst. of Physics, New York, 1995), p. 95 [AIP Conf. Proc. **356**, 95 (1995)].
61. I. A. Grishaev and N. N. Nasonov, Pis'ma Zh. Tekh. Fiz. **3**, 1084 (1977) [Sov. Tech. Phys. Lett. **3**, 446 (1977)].
62. T. Tajima and M. Covenago, Phys. Rev. Lett. **59**, 1440 (1987).

63. T. Tajima and B. S. Newberger, Phys. Rev. A **40**, 6897 (1989).
64. S. C. Kitson, W. L. Barnes, and J. R. Sambles, Phys. Rev. Lett. **77**, 2670 (1996).
65. *Discrete Simulation of Plasma*, Ed. by Yu. S. Sigov (Inst. Prikl. Mat. im. M. V. Keldysha Akad. Nauk SSSR, Moscow, 1990).
66. O. V. Batishchev, V. I. Karas', V. D. Levchenko, and Yu. S. Sigov, Fiz. Plazmy **20**, 654 (1994) [Plasma Phys. Rep. **20**, 587 (1994)].
67. O. V. Batishchev, Preprint No. 4, IPM im. M.V. Keldysha AN SSSR (Keldysh Institute of Applied Mathematics, Academy of Sciences of USSR, Moscow, 1986).
68. O. V. Batishchev and Yu. S. Sigov, Preprint No. 87, IPM im. M.V. Keldysha AN SSSR (Keldysh Institute of Applied Mathematics, Academy of Sciences of USSR, Moscow, 1988).
69. N. I. Aœzatskiœ, E. Z. Biller, V. N. Boriskin, *et al.*, Fiz. Plazmy **20**, 671 (1994) [Plasma Phys. Rep. **20**, 603 (1994)].
70. *Theory and Computation of Linear Accelerators: Collection of Articles* (Gosatomizdat, Moscow, 1962).
71. B. M. Bolotovskiœ and S. N. Stolyarov, *Einsteinian Collection, 1978-1979*, Ed. by V. L. Ginzburg *et al.* (Nauka, Moscow, 1983), p. 173.
72. V. P. Kovalenko, V. M. Pergamenshchik, and V. I. Starkov, Fiz. Plazmy **11**, 417 (1985) [Sov. J. Plasma Phys. **11**, 241 (1985)].
73. V. M. Kuklin, Ukr. Fiz. Zh. **31**, 853 (1986).
74. A. Ts. Amatuni, E. V. Sekhposyan, and S. S. Elbakyan, Fiz. Plazmy **12**, 1145 (1986) [Sov. J. Plasma Phys. **12**, 662 (1986)].
75. V. A. Balakirev, Yu. P. Bliokh, I. N. Onishchenko, and Ya. B. Fainberg, Fiz. Plazmy **14**, 218 (1988) [Sov. J. Plasma Phys. **14**, 123 (1988)].
76. S. V. Bulanov, L. M. Kovrizhnykh, and A. S. Sakharov, Phys. Rep. **186**, 1 (1990).
77. A. V. Kuznetsov, T. Zh. Esirkepov, F. F. Kamenets, and S. V. Bulanov, Fiz. Plazmy **27**, 225 (2001) [Plasma Phys. Rep. **27**, 211 (2001)].
78. I. V. Karas', Author's Abstract of Candidate's Dissertation in Physics and Mathematics (NNTs KhFTI, Khar'kov, 2001).
79. J. B. Rosenzweig, Phys. Rev. A **38**, 3634 (1998).
80. P. Schoessow, E. Chojnacki, W. Gai, *et al.*, in *Proceedings of the 1993 Particle Accelerator Conference, Washington, 1993*, Vol. 4, p. 2596.
81. N. Barow and J. Rosenzweig, in *Proceedings of the 1995 Particle Accelerator Conference, Dallas, 1995*, Vol. 2, p. 976.
82. N. Barow, M. Conde, J. B. Rosenzweig, *et al.*, in *Proceedings of the 1995 Particle Accelerator Conference, Dallas, 1995*, Vol. 2, p. 631.
83. M. I. K. Santala, Z. Najmudin, E. L. Clark, *et al.*, Phys. Rev. Lett. **86**, 1227 (2001).
84. C. Coverdale, C. B. Darrow, C. D. Decker, *et al.*, in *Proceedings of the 1995 Particle Accelerator Conference, Dallas, 1995*, Vol. 2, p. 758.
85. R. Vagner, S.-Y. Chen, A. Maksimchuk, and D. Umstadter, Phys. Rev. Lett. **78**, 3125 (1997).
86. J. Rosenzweig, D. Cline, B. Cole, *et al.*, Phys. Rev. Lett. **61**, 98 (1988).
87. A. Ogata, in *Proceedings of the Congress on Advances Accelerator Concepts, Port Jefferson, 1992*, Ed. by J. Wurtele (American Inst. of Physics, New York, 1993), p. 420 [AIP Conf. Proc. **279**, 420 (1993)].
88. J. M. Dawson, Phys. Plasmas **6**, 4436 (1999).
89. C. E. Clayton, K. A. Marsh, M. Everett, *et al.*, Phys. Rev. Lett. **70**, 137 (1993).
90. C. E. Clayton, K. A. Marsh, M. Everett, *et al.*, in *Proceedings of 1993 Particle Accelerator Conference, Washington, 1993*, Vol. 4, p. 2551.
91. A. G. Litvak, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **7**, 562 (1964).
92. M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972).
93. G. Shvets and J. S. Wurtele, Phys. Rev. Lett. **73**, 3540 (1994).
94. C. I. Moore, A. Ting, K. Krushelnick, *et al.*, Phys. Rev. Lett. **79**, 3909 (1997).
95. K. Krushelnik, A. Ting, C. I. Moore, *et al.*, Phys. Rev. Lett. **78**, 4047 (1997).
96. C. E. Clayton, K.-C. Tzeng, D. Gordon, *et al.*, Phys. Rev. Lett. **81**, 100 (1998).
97. F. Dorchies, F. Amiranoff, V. Malka, *et al.*, Phys. Plasmas **6**, 2903 (1999).
98. F. Amiranoff, S. Baton, D. Bernard, *et al.*, Phys. Rev. Lett. **81**, 995 (1998).
99. N. E. Andreev, V. I. Kirsanov, and A. S. Sakharov, Fiz. Plazmy **26**, 416 (2000) [Plasma Phys. Rep. **26**, 388 (2000)].
100. S. Y. Chen, M. Krishnan, A. Maksimchuk, *et al.*, Phys. Plasmas **6**, 4739 (1999).
101. V. Malka, N. Renard-Le Galloudec, S. Huller, *et al.*, Phys. Plasmas **7**, 4259 (2000).
102. G. S. Sarkisov, V. Y. Bychenkov, V. N. Novikov, *et al.*, Phys. Rev. E **59**, 7042 (1999).
103. J. Faure, V. Malka, J.-R. Marques, *et al.*, Phys. Plasmas **7**, 3009 (2000).
104. P. E. Young and P. R. Bolton, Phys. Rev. Lett. **77**, 4556 (1996).
105. B. Ritchie and P. R. Bolton, Phys. Rev. E **58**, 6460 (1997).
106. M. Tatarakis, J. R. Davies, P. Lee, *et al.*, Phys. Rev. Lett. **81**, 999 (1998).
107. M. Borghesi, A. J. Mackinnon, A. R. Bell, *et al.*, Phys. Rev. Lett. **81**, 112 (1998).
108. G.-Z. Sun, E. Ott, Y. C. Lee, and P. Guzdar, Phys. Fluids **302**, 526 (1987).
109. N. E. Andreev and L. M. Gorbunov, Usp. Fiz. Nauk **169**, 53 (1999).
110. N. Barow, M. Conde, W. Gai, and J. B. Rosenzweig, IEEE Trans. Plasma Sci. **PS-26**, 627 (1998).

Translated by O. E. Khadin

SPELL: emittance