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Numerical Modeling of the Formation of Steady-State Nonequilibrium Distributions of Particles Interacting through a Power-Law Potential

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Abstract—The formation of a steady-state nonequilibrium distribution function of particles interacting through the repulsive potential $U \sim \alpha/r^{\beta}$ $(1 \le \beta \le 4)$, which operates at an infinite range, is studied numerically. The collisional particle dynamics in such a system is investigated using a spatially homogeneous nonlinear collision integral in the Landau–Fokker–Planck form, which is a model Boltzmann collision integral for arbitrary potentials of interaction accompanied by little momentum transfer between particles in collisions. Numerical modeling is based on completely conservative difference schemes. It is shown that the principal condition for the existence of steady-state nonequilibrium distributions is the presence of a particle or an energy flux oriented in the proper manner in momentum space. A steady-state local distribution exists inside the momentum interval between the energy source and sink and has the form of a gradually decreasing function. Since a radical change in the distribution function under nonequilibrium conditions leads to an anomalous enhancement of the conduction of a medium and its emission characteristics, the results obtained can be used, e.g., to predict the behavior of semiconductors with an intrinsic or extrinsic conductivity under the action of particle fluxes or electromagnetic radiation. *© 2002 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Nonequilibrium states of various physical systems are attracting increased interest in connection with the development and wide use of high-power particle and energy sources.

In physical systems in which the interactions between particles or waves can be described by the kinetic equations for waves, quasi-particles, or particles, the problem of constructing steady-state nonequilibrium distributions reduces to that of solving the kinetic equations. In this case, the local character of the steady-state nonequilibrium distribution corresponds to the convergence of the collision integral (at each point in momentum space, the main contribution to the collision integral comes from collisions between particles with close momenta).

For a classical (nondegenerate) gas, the Boltzmann kinetic equation has the form

$$\frac{\partial f(\mathbf{p})}{\partial t} = \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 W(\mathbf{p}, \mathbf{p}_1 | \mathbf{p}_2, \mathbf{p}_3) \times [f(\mathbf{p}_2) f(\mathbf{p}_3) - f(\mathbf{p}) f(\mathbf{p}_1)] \times \delta(E + E_1 - E_2 - E_3)\delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3), \quad (1)$$

where $W(\mathbf{p}, \mathbf{p}_1 | \mathbf{p}_2, \mathbf{p}_3)$ is the transition probability due to collisions, $f(\mathbf{p})$ is the particle (electron) distribution function, $\mathbf{p}_i$ and E_i are the momentum of an electron and its energy, and $\delta(x)$ is the Dirac delta function.

The distribution function with which the collision integral vanishes at any time t satisfies the condition

$$f(\mathbf{p}_2) f(\mathbf{p}_3) - f(\mathbf{p}) f(\mathbf{p}_1) = 0 \quad (2)$$

and is a steady solution to Eq. (1). Condition (2) with the energy and momentum conservation laws leads to a thermodynamically equilibrium Maxwellian (or Fermi–Dirac) distribution function f_{Maxw}.

At the beginning of this paper, we briefly review the results concerning the existence of steady-state local isotropic nonequilibrium distributions in a solid-state plasma, which seems to be a convenient object to study because it provides the possibility of controlling the extent to which the system in a stable state is nonequilibrium. The existence of a steady-state nonequilibrium distribution implies that there are particle source and sink in momentum space. The energy (particle) source and sink can be provided by ion beams, high-power laser radiation, emission currents, fluxes of charged particles produced in fusion or fission reactions, etc.

The characteristic feature of systems of charged particles interacting through the Coulomb potential is that the interaction cross section diverges in the limit of a small momentum transferred in a collision. To describe a gas or a semiconductor plasma for which the Cou-

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lomb logarithm is large ($\ln\Lambda = 10$–15), it is sufficient to expand the integrand in powers of the small momentum transferred (the diffusion approximation) and to represent the collision integral in the Landau or Fokker–Planck form [1, 2]. The Landau (Fokker–Planck) kinetic equation, which is an essential element in various physical models, is widely applied in plasma physics. In Section 3, the kinetic equations mentioned above are presented and the formulation of the problem is discussed in detail. The formation of a steady-state nonequilibrium particle distribution function is investigated numerically for a wide range of parameters (different source and sink intensities, localization of the source and sink in momentum space, etc.). The calculated results are discussed, and it is shown that the Landau collision integral provides a correct description of a nonequilibrium distribution function in a plasma in which the main process is a small-angle scattering with little momentum transfer between the particles.

The objective of Section 4 is to consider the features of the conduction and emission characteristics of metals and semiconductors under the action of intense particle fluxes or laser light. As was mentioned in [3], the photoconductivity of a semiconductor irradiated by light whose frequency ω is insufficient to cause the interband transition $\hbar\omega < V_g$ (where V_g is the width of the forbidden band) may become anomalously high because of the formation of a nonequilibrium distribution of electrons and holes. Based on an analysis of the experimental results, Aseevskaya *et al.* [4] arrived at the conclusion that the irradiation of semiconductors by α-particles may substantially change their conduction properties. Some experimental data on the emission current from metals irradiated by lasers (in particular, its dependence on the retarding potential) cannot be explained on the basis of equilibrium distribution functions [5–7], but they are reasonably explicable in terms of the mechanism for the formation of nonequilibrium distributions [6–8].

In our opinion, all these circumstances confirm the above general statement about the formation of nonequilibrium distributions in the presence of external pumping. In this context, the physical effect associated with such fundamental changes in the electron distribution function are discussed.

2. STEADY-STATE NONEQUILIBRIUM ELECTRON DISTRIBUTION WITH AN ENERGY FLUX ALONG THE SPECTRUM

In the theory of the turbulence of incompressible liquids, a universal (independent of the structures of the source and sink) steady-state nonequilibrium energy spectrum ε_k in the wavenumber range between the wavenumbers of the excited and efficiently damped motions was first obtained by Kolmogorov [9]. The well-known Kolmogorov energy spectrum ε_k of the hydrodynamic turbulence has the form

$$\varepsilon_k = AP_1^{2/3}k^{-11/3}, \tag{3}$$

where A is a constant and P_1 is the energy flux along the spectrum. Spectrum (3) was derived on the basis of the hypothesis about the local character of turbulence, i.e., under the assumption that the interaction of perturbations occurring on close spatial scales is the strongest. In the theory of weak turbulence, universal wave spectra in the form of solutions to equations with the wave–wave collision integral were first derived by Zakharov [10].

Universal steady-state nonequilibrium power-law particle distribution functions $f = Ap^{2s}$ that are exact solutions to kinetic equations with the Boltzmann collision integral were first obtained by Kats *et al.* [11]. In order for such distributions to form, there should be a source and sink of particles (or energy) that provide a constant particle (or energy) flux in momentum space.

When the transition probability is an nth-power homogeneous function of momentum, the power s can naturally be determined by expressing the integrand in terms of the variables $\mathbf{p}_i/p$. Then, the right-hand side of Eq. (1) reduces to the product of the factor p^{4s+n+4} and an integral independent of p. In order to determine the particle (I_0) and energy (I_1) fluxes in momentum space, we use the following equations, which relate them to the collision integral:

$$\nabla \cdot \left(j_i(p)\frac{\mathbf{p}}{p}\right) = -E^i\left(\frac{\partial f}{\partial t}\right)_{St}, \tag{4}$$

where $I_i = 4\pi p^2 j_i$ and E is the particle energy. Solving Eqs. (4), we obtain

$$I_i = A^2\alpha^{1-i}\frac{R(s,n)}{4s+n+9+2(i-1)}p^{4s+n+9+2(i-1)}, \tag{5}$$

where α = const and the collision integral contains the factor $R(s, n)$.

From expressions (5), we can see that, if the powers s_i satisfy the conditions

$$\gamma = 4s+n+9+2(i-1) = 0, \quad i = 0, 1, \tag{6}$$

then the fluxes I_i in momentum space are either constant or are equal to zero when $R(s, n)$ has a first-order root at $s = s_i$. In both cases, the collision integral vanishes. In the equilibrium situation, the fluxes obtained from Eqs. (4) are identically zero, because the integrand in spectrum (2) with an equilibrium distribution vanishes at each point p. However, for a nonequilibrium distribution function, there is no need in conditions (6) and only the integral of the collision operator (rather than the operator itself) vanishes.

The distribution function Ap^{2s} corresponds to a steady-state nonequilibrium situation with a constant particle (or energy) flux in momentum space. The direction of the flux is determined by the sign of the derivative $dR/d\gamma$ at $\gamma = 0$, and A is described by the expression

$$A^2 = I_i \alpha^{i-1} \lim_{\gamma \to 0} \left| \frac{dR}{d\gamma} \right|^{-1}. \quad (7)$$

The collision integral for steady-state nonequilibrium distributions satisfying the above conditions was calculated in a straightforward way in [12, 13]. In those papers, it was shown that the powers s of the local nonequilibrium particle distribution functions (for which the collision integral converges) lie within the intervals

$$-\frac{3}{2} < s_0 < -1; \quad -\frac{3}{2} < s_1 < -\frac{5}{4}; \quad (8)$$

The powers 0 and 1 correspond to the fluxes I_0, I_1 = const, respectively.

In accordance with conditions (6), intervals (8) correspond to the following intervals of the powers of a homogeneous function describing the transition probability:

$$-3 < n < -1, \quad I_0 = \text{const}, \quad (9)$$

$$-4 < n < -3, \quad I_1 = \text{const}. \quad (10)$$

From inequalities (9) and (10), we can see that, in the case of Coulomb interaction ($n = -4$), the collision integral diverges, which corresponds to the well-known singularity of W when the momentum transferred in collisions is small. Note that, in this case, the interaction potential in the coordinate (rather than momentum) space has the form $U = \frac{\text{const}}{r^{-4/n}}$. According to [12, 13], the divergence at $n = -4$ can be eliminated by taking into account the Debye screening. Thus, in [12, 13], it was shown that, for the transition probability corresponding to the screened Coulomb potential $W = 2e^4/(q^2 + a_i^2)^2$ (where q is the momentum transferred in a collision and a_i is the Debye momentum) and for particles with the momenta $p \gg a_i$, taking into account the Debye screening, on the one hand, eliminates the Coulomb divergence and, on the other, does not change the power of the steady-state nonequilibrium particle distribution function with a constant energy flux in momentum space. The power of the distribution function corresponds to the asymptotic behavior of W with the power $n = -4$.

Hence, in [12, 13], it was shown that particles with the momenta $p \gg a_i$ obey a local (in the sense of the convergence of the collision integral) power distribution function, in which case the particle density is determined by the flux intensity in momentum space. The flux conservation is ensured by the source and sink, whose positions should be consistent with the direction of the flux.

The case considered in [14] is more typical of solid-state plasmas; specifically, the electron distribution function is a power function inside the interval (pp'', p') between the localized energy source and sink in momentum space and is a thermodynamically equilibrium Fermi–Dirac function outside this interval. It was shown that, under certain conditions on the positions of the source and sink and on their intensities (see below), the steady-state nonequilibrium electron distribution function is close to a universal function ($s = -5/4$ in the case of an unbounded inertial interval). Thus, the power s of the distribution of the filling numbers N_s differs from –5/4 by less than 10% under the conditions

$$|p'' - p'| \approx (5\text{–}6)p_{ch}, \quad N_s(p) \gg 10^{-3}, \quad p_{ch} = (2\text{–}3)a_i. \quad (11)$$

Hence, the electrons may obey a universal nonequilibrium distribution even when the filling numbers are substantially (by one to two orders of magnitude) smaller than the equilibrium numbers.

We have considered the formation of steady-state nonequilibrium distributions with sources and sinks localized in momentum space. Note, however, that it is often necessary to deal with systems in which the source and/or sink are nonlocalized; in particular, ionization by the wake field is described by a nonlocal source in momentum (energy) space.

3. NUMERICAL MODELING OF THE FORMATION OF STEADY-STATE NONEQUILIBRIUM PARTICLE DISTRIBUTIONS

In what follows, we will investigate a spatially homogeneous isotropic gas consisting of a single particle species. The particles are assumed to interact through the power-law potential $U = \frac{\text{const}}{r^\beta}$, where $\beta = -4/n$. Our model is based on the Boltzmann equation with the Landau collision integral [15, 16]. The first three tensor moments and the first four scalar moments of the model collision integral are assumed to coincide with those of the exact collision integral. The particle number and the energy are conserved, and the H-theorem for the Boltzmann equation is satisfied. The equations with the model collision integral provide a correct description of the system in the 20-moment approximation of the Grad method. Finally, an exact solution to the Landau equation for Maxwellian molecules ($\beta = 4$) is the exact solution to the Boltzmann equation.

We will be considering potentials operating at an infinite range ($1 \le \beta \le 4$), in which case a local nonequilibrium particle distribution may form [see inequalities (9) and (10)]. Note that the dynamics of the particles interacting by means of the Coulomb potential ($\beta = 1$) was previously studied using both the Landau kinetic equation and the Fokker–Planck kinetic equation.

For an isotropic distribution function $f(v, t)$, the kinetic equation with the Landau collision integral has the form

$$\frac{\partial f}{\partial t} = I_L[f, f]$$

$$= \frac{\Gamma}{v^2}\frac{\partial}{\partial v}\left\{\frac{1}{v}\int_0^\infty dw Q(v, w)\left[wf(w)\frac{\partial f(v)}{\partial v} - vf(v)\frac{\partial f(w)}{\partial w}\right]\right\},$$

$$Q(v, w) = \frac{a(v, w)(v+w)^{\eta+4} + b(v, w)|v-w|^{\eta+4}}{(\eta+4)(\eta+2)(\eta+6)},$$

$$0 \le v, \quad w < \infty, \quad t \ge 0, \qquad (12)$$

$$a(v, w) = [(\eta+4)vw - (v^2 + w^2)],$$

$$b(v, w) = [(\eta+4)vw + (v^2 + w^2)], \quad \eta = (\beta - 4)/\beta;$$

where $\Gamma = \frac{16\pi^2 e^4 \ln\Lambda}{m^2}$ and the symmetric kernel $Q(v, w) = Q(w, v)$ for the Coulomb potential is described by the expressions $Q(v, w) = (2/3)w^3$ for $w \le v$ and $Q(v, w) = (2/3)v^3$ for $w \ge v$.

Kinetic equation (12) with the Landau collision integral has an equilibrium solution in the form of a Maxwellian distribution function:

$$f_{\mathrm{Maxw}} = \frac{n_p}{\pi^{3/2}} v_T^{-3} \exp\left[-\frac{v^2}{v_T^2}\right], \quad v_T = \sqrt{\frac{2k_B T}{m}},$$

where k_B is Boltzmann's constant and T is the temperature.

In the absence of sinks (or sources), the number of particles in the system and its energy are both constant in time:

$$n_p = 4\pi\int_0^\infty f(v, t)v^2 dv = \mathrm{const},$$

$$n_p E = 2\pi m\int_0^\infty f(v, t)v^4 dv = n_p\frac{3}{2}k_B T = \mathrm{const}.$$

The distribution function $f(v, t)$ is finite at $v = 0$ and decreases fairly rapidly as $t \longrightarrow \infty$ and $v \longrightarrow \infty$.

We introduce dimensionless variables such that the velocity is expressed in units of the thermal velocity $V_T = (3/2)^{1/2}v_T$ and the time is measured in units of the electron–electron relaxation time, which has the form $\tau_{ee} = \frac{v_T^3 m^2}{4\pi n_p e^4 \ln\Lambda}$ in the case of Coulomb interaction. In these variables, the parameters of the normalized dimensionless distribution function are $n_p = 1$, $v_T = 1$, $E = 3m/4$, in which case the constant Γ in kinetic equation (12) is equal to unity.

Note that rarefied collisional plasmas ($\beta = 1$) under laboratory conditions and in astrophysical applications are usually simulated using kinetic equations with the Fokker–Planck collision integral:

$$\frac{\partial f}{\partial t} = I_{FP}[f, f] = \frac{1}{v^2}\frac{\partial}{\partial v}\left[A(v)\frac{\partial f}{\partial v} + B(v)f(v, t)\right],$$

$$0 \le v < \infty, \quad t \ge 0.$$

For numerical modeling, it is most convenient to write the Fokker–Planck equation in symmetric form [17]:

$$\frac{\partial f}{\partial t} = I_{FP}[f, f]$$

$$= \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\frac{1}{v}\frac{\partial}{\partial v}\int_0^v [f(v)P(x) - f(x)P(v)]x^2 dx\right\}; \qquad (13)$$

where $P(x) = 2\int_v^\infty f(x)x dx$.

In modeling the formation of a nonequilibrium distribution on the basis of kinetic equations (12) and (13) with the Landau and Fokker–Planck modified collision integrals, respectively, the particle and energy fluxes in momentum space were taken into account by supplementing the right-hand sides of the kinetic equations with the following terms, which describe the energy (particle) source and sink:

$$\frac{\partial f}{\partial t} = I_{FP, L}[f, f] + S_+ - S_-.$$

The source and sink functions were modeled by exponential functions localized in different intervals in momentum space, $S_\pm \sim I_\pm \exp\{-\alpha_1(v - v_\pm)^2\}$, and by the localized function (Dirac delta function) $S_\pm \sim I_\pm\delta(v - v_\pm)/v^2$ or

$$S_\pm \sim I_\pm \frac{\delta(v - v_\pm)}{v^2} f(v_-, t). \qquad (14)$$

When $I_+ = I_-$, we deal with the energy flux from the source toward the sink, and, when $I_+ = I_-\frac{v_-^2}{v_+^2}$, we deal with an analogous particle flux. The direction of the fluxes depends on the ratio of the velocities v_- and v_+.

The initial distribution function was assumed to be either a Maxwellian function or a delta function.

The simulations were carried out on the basis of completely conservative implicit difference schemes [15, 17] for which discrete analogues of the conservation laws are satisfied and which make it possible to perform long-run simulations without accumulating computational errors. An infinite velocity interval was bounded from above by the highest possible velocities $10v_T$–$14v_T$, at which the distribution function was equal to a computer zero. The initial distribution in the form of a delta function was modeled by the function $\delta(v - v_0)$, which was zero over the entire velocity range used in simulations, except for a single velocity value (usually, $v_0 = 1$).

Since the relaxation problem is, in a sense, a test problem, we began by considering the Cauchy problem for kinetic equations with the initial distribution function $f^0(v) = \delta(v - 1)/v^2$. The corresponding simulations were carried out for kinetic equations (12) and (13) with the Landau and Fokker–Planck collision integrals.

Now, we proceed to a discussion of the results obtained from simulations of the formation of steady-state nonequilibrium distributions in the presence of energy or particle fluxes in momentum space. The right-hand sides of kinetic equations (12) and (13) were supplemented with the source and sink terms S_+ and S_-. First, we consider the results obtained for the case in which the source and sink were consistent with the direction of the collision-related flux in momentum space. Note that an analytic solution of the kinetic equations with a localized source and sink yields a correct flux direction, namely, the flux is directed downward along the velocity axis. From Fig. 1, we can see that, in the presence of an energy flux in momentum space, the initial particle distribution in the interval between the source and sink relaxes to a Kolmogorov steady-state nonequilibrium distribution, while the distribution function outside this interval is in thermodynamic equilibrium. Recall that the positions of the source and sink should be consistent with the flux direction in momentum space. In order to convince ourselves that this condition is very important, we interchanged the source and sink in energy space and performed the corresponding simulations. The results calculated for reversed positions of the source and sink are illustrated in Fig. 2, which shows the logarithm of the distribution function versus the square of the dimensionless velocity. In this case, a change in the flux intensity by several orders of magnitude produces no change in the equilibrium particle distribution.

Figure 3 was calculated for the source and sink terms described by the exponential functions of velocity. The source S_+ is localized in a "narrow" interval of energies corresponding to seven thermal velocities, and the sink S_- is localized in the energy interval corresponding to four thermal velocities. The lengths of the intervals in which the source and sink are localized is controlled by the coefficient α_1 in the exponential functions. In the case at hand, the coefficient is equal to 100; this very large value ensures a strong localization of the source and sink.

In order to investigate how the electron distribution function depends on the extent to which the source and sink are localized in energy space, we carried out a series of computations with a smaller (10 instead of 100) value of the above coefficient in the exponential functions. A comparison between the results presented in Figs. 3 and 4 shows that the character of the steady-state nonequilibrium distribution in the main region between the source and sink is independent of the

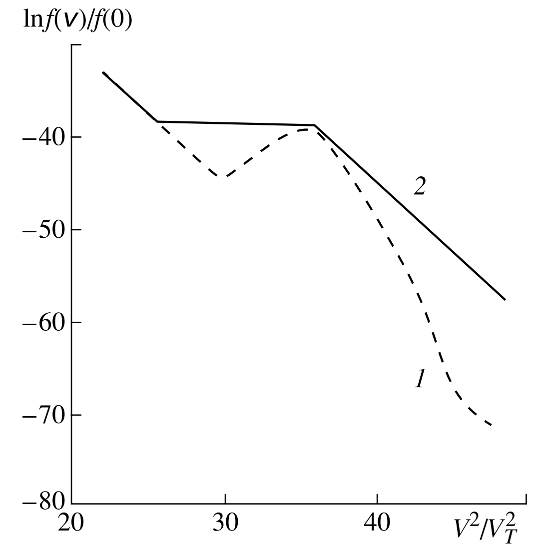

Fig. 1. Logarithm of the steady-state nonequilibrium distribution function normalized to its value at $v = 0$ vs. squared velocity. The computations were carried out for the initial Maxwellian distribution function and the source function with $I = 10^{-16}$, $v_- = 5$, and $v_+ = 6$. The solid and dashed curves were calculated from the Fokker–Planck equation at the times $t_1 = 5$ and $t_2 = 100$, respectively.

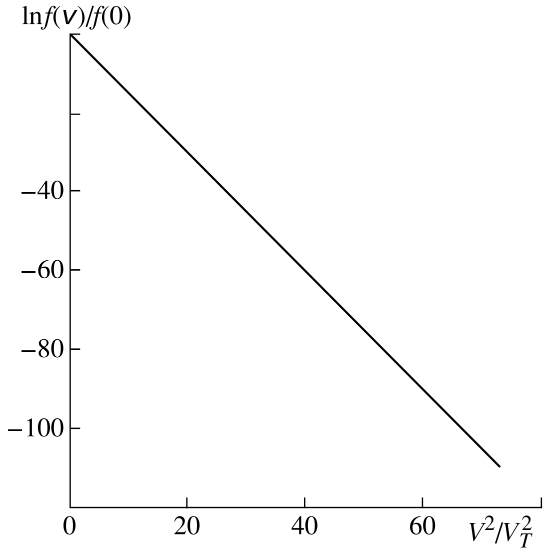

Fig. 2. Steady-state (equilibrium) distribution functions calculated for reversed positions of the source and sink ($v_+ = 5$, $v_- = 7$) from the Fokker–Planck equation and from the Landau equation with $\beta = 1$, 2, and 3.

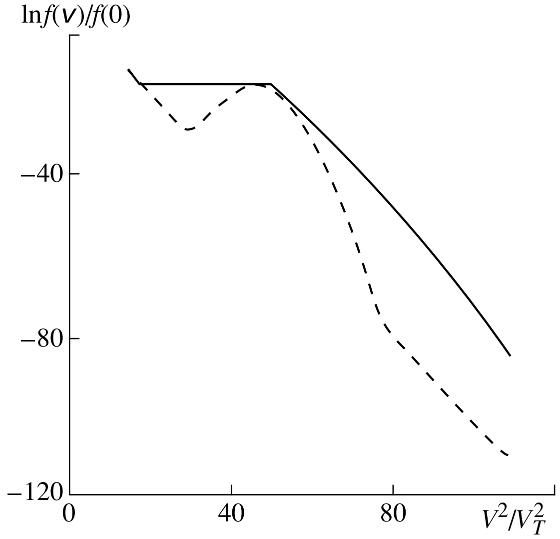

Fig. 3. Distribution function calculated from the Fokker–Planck equation with the source and sink functions $S_{\pm} \sim I_{\pm}\exp\{-\alpha_1(v - v_{\pm})^2\}$ for $\alpha_1 = 100$, $v_- = 4$, and $v_+ = 7$. The dashed and solid curves refers to the times $t = 25$ and 100, respectively.

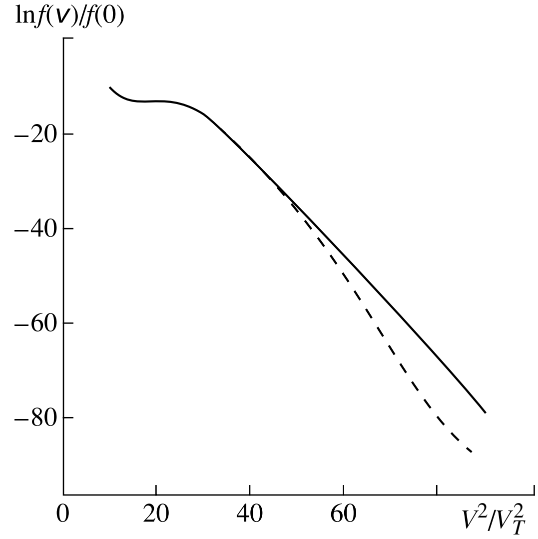

Fig. 4. Distribution function calculated from the Fokker–Planck equation with the source and sink functions $S_{\pm} \sim I_{\pm}\exp\{-\alpha_1(v - v_{\pm})^2\}$ for $\alpha_1 = 10$, $v_- = 3$, and $v_+ = 5$. The dashed and solid curves refers to the times $t = 25$ and 100, respectively.

extent to which they are localized, thereby providing evidence of the local (universal) character of this solution to the kinetic equation.

Figure 5 shows how the distribution function depends on velocity for different flux intensities. We established that, when the intensities of the source (I_+) and sink (I_-) are both low, a universal nonequilibrium distribution forms in the velocity range $v \le v_+$. The reasons for this are twofold: first, with increasing velocity, the cross section for Coulomb scattering decreases (in proportion to $\sim v^{-3}$) and, second, diffusion due to Coulomb collisions always gives rise to energy and particle fluxes toward the region of the main ("background") equilibrium distribution function. As a result, the higher the intensities of the source and sink, the lower the relative intensity of the particle flux toward the background plasma and, accordingly, the larger the region in which a universal nonequilibrium particle distribution forms between the source and sink. It should be noted that, as the flux becomes more intense, the values of a nonequilibrium distribution function increase. This is because the nonequilibrium distribution function is proportional to the flux intensity [see Eq. (7)]. Some values of the steady-state nonequilibrium distribution function that were obtained by solving the Landau equation numerically are presented in Table 1. These values illustrate the results of a detailed study of the dependence of the solution on the flux intensity in momentum space, which was varied over a wide range. The initial source and sink functions were taken to be functions (14) with $v_- = 4$, $v_+ = 8$, and $dE = 0$.

Now, we consider the formation of a nonequilibrium particle distribution in the case of a constant particle ($dE = 0$, $I_+ = I_-\frac{v_-^2}{v_+^2}$) or energy ($dN = 0$, $I_+ = I_-$) flux. In order to simplify comparisons of the results obtained, it is convenient to normalize the distribution function to

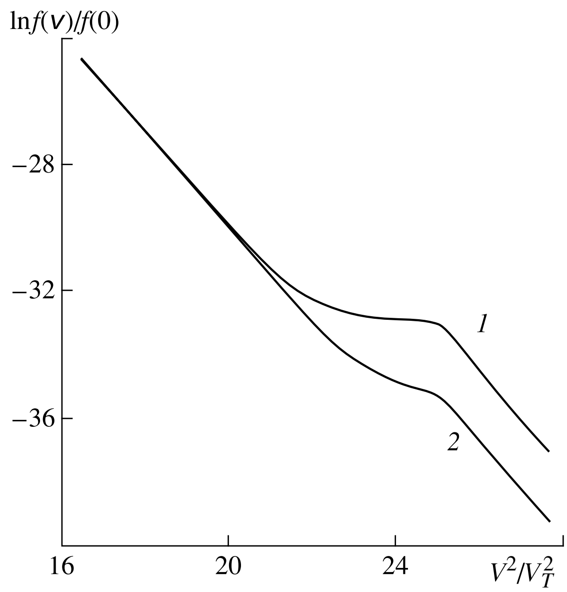

Fig. 5. Steady-state distribution function calculated from the Landau equation with $\beta = 2$ for the source and sink functions in the form of δ functions (14) at different flux intensities $I_1 = 0.01$ and $I_2 = 0.001$ ($v_- = 4$, $v_+ = 5$).

its value at zero, as is done in all figures presented in this paper, and, in particular, in Fig. 6, from which we can see the formation of gradually decreasing distribution functions with close powers s in the cases of a particle and an energy flux.

Let us analyze the shapes of the distribution functions of the particles interacting by means of different potentials with powers in the range $1 \le \beta \le 4$. Note that the power $\beta = 1$ corresponds to the Coulomb interaction potential, the power $\beta = 2$ refers to the dipole interaction potential, and the power $\beta = 4$ describes the interaction between the so-called Maxwellian molecules.

The dependence of the distribution function $f(v_+)$ on the power β for $I = 0.01$ is illustrated in Table 2.

Figure 7 shows nonequilibrium distribution functions calculated for the interaction potentials with β = 1, 2, and 4 and for an energy flux with the constant intensity $I = 0.01$. As may be seen, the powers s of the distribution functions that form in these three cases are close to each other, in agreement with conditions (8). The larger the power β, the shorter the interval over which the distribution function is nonequilibrium. These calculated results agree qualitatively with the above analytic predictions [see expression (7) and conditions (8)].

4. MECHANISM FOR THE FORMATION OF THE ELECTRON DISTRIBUTION FUNCTION IN THE INTERACTION OF ELECTROMAGNETIC RADIATION WITH A SOLID-STATE PLASMA

In this section, we focus on the distinctive features of the conduction and emission characteristics of a semiconductor plasma that is affected by intense particle fluxes or laser radiation.

Let us compare the characteristic times of the electron energy relaxation due to electron–electron and electron–phonon collisions. In the interaction of particle fluxes or fluxes of intense electromagnetic radiation with a semiconductor plasma, ionization processes produce electrons with the energies $E \ge \hbar\omega$, where ω is the radiation frequency. The energy spectrum of the electrons produced by particle fluxes extends from tens of electronvolts to tens of kiloelectronvolts. According to [18], at high temperatures $T > \Theta_D$ (where Θ_D is the Debye temperature), the frequency of collisions between electrons with sufficiently high energies E ($E \gg k_BT$) is described by the expression

$$\gamma^{ee}(E, T) = \gamma_0^{ee}(T)\left[1 + \left(\frac{E}{k_BT}\right)^2\right], \qquad (15)$$

where $\gamma_0^{ee}(T)$ is the classical high-temperature electron–electron collision frequency, which is proportional to T^2. Under the same conditions, the electron–phonon collision frequency has the form [18]

$$\gamma^{ef} = \frac{f(\Theta_D)T}{\Theta_D}, \qquad (16)$$

where $f(\Theta_D)$ is the classical high-temperature electron–phonon collision frequency at $T = \Theta_D$. For the processes

Table 1

I	$f(3.95)$	$f(7.95)$	$f(8)$
10	0.393×10^{-9}	0.417×10^{-10}	0.417×10^{-10}
1	0.475×10^{-9}	0.144×10^{-10}	0.144×10^{-10}
0.1	0.508×10^{-9}	0.189×10^{-11}	0.189×10^{-11}
0.01	0.517×10^{-9}	0.197×10^{-12}	0.197×10^{-12}
0.001	0.518×10^{-9}	0.198×10^{-13}	0.198×10^{-13}
0.0001	0.519×10^{-9}	0.196×10^{-14}	0.196×10^{-14}

Note: The values of the distribution function are seen to increase with increasing intensity I: at low intensities (to 0.1), the values of the distribution function increase in proportion to I because of the large contribution of the interaction of nonequilibrium particles (i.e., the particles from the interval between the source and sink) with the background particles, whose distribution is thermodynamically equilibrium; at moderate intensities (from 0.1 to 20), the distribution function is of a universal nature over the entire interval between the source and sink and is proportional to the square root of the intensity I, in agreement with expression (7); and, at higher intensities, the distribution function is no longer proportional to the square root of the intensity because it enters into the sink function in formulas (14).

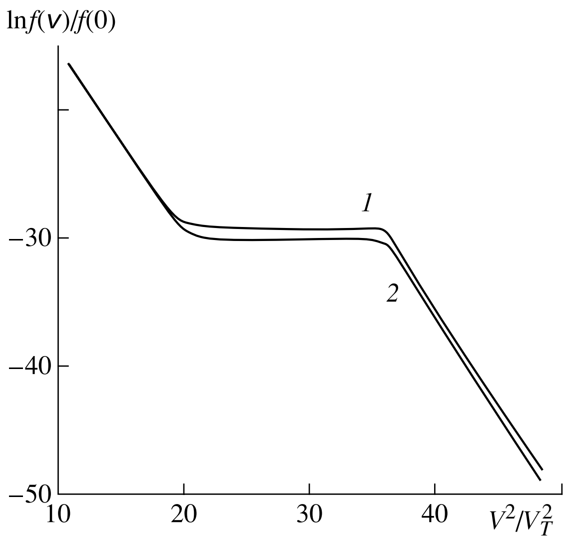

Fig. 6. Steady-state distribution function calculated from the Landau equation with $\beta = 1$ for the source and sink functions in the form of δ functions (14) at $I_{\mp} = 0.01$, $v_- = 4$, and $v_+ = 6$. Curve *1* and *2* correspond to a constant energy flux ($dN = 0$) and a constant particle flux ($dE = 0$), respectively,

Table 2

β	1	1.5	2	3	4
$f(8)$	0.197×10^{-12}	0.123×10^{-13}	0.266×10^{-14}	0.771×10^{-15}	0.384×10^{-15}

under discussion, the conditions under which expressions (15) and (16) are valid hold because we have $E >$ 10 eV, T = 300 K, and Θ_D = 200–300 K. Note that the quantities that are inversely proportional to the collision frequencies (15) and (16) generally differ from the electron energy relaxation times, which depend, in particular, on the number of collisions in which an electron loses its total energy E; therefore, it is necessary to take into account the factor $\xi = E/E_1$ (where E_1 is the energy lost by an electron in a single collision). According to the results obtained in [18], in the case in question, we can write

$$\gamma^{\rm ef} \approx \gamma_0^{ee}, \quad \gamma^{ee} \gg \gamma_0^{ee},$$

On the contrary, for electron–electron collisions, the factor ξ can be on the order of unity, and, for electron–phonon collisions, we have $E/k_B\Theta_D > 3 \times 10^2$. Hence, in this case, the time of the electron energy relaxation due to electron–electron collisions is substantially shorter than that due to electron–phonon collisions.

A comparison of the characteristic ionization time with the relaxation times shows that, in our case, the electron distribution function is quasi-steady and is governed primarily by electron–electron collisions. Consequently, this function can be determined from the condition for the Boltzmann (Landau–Fokker–Planck) collision integral to vanish.

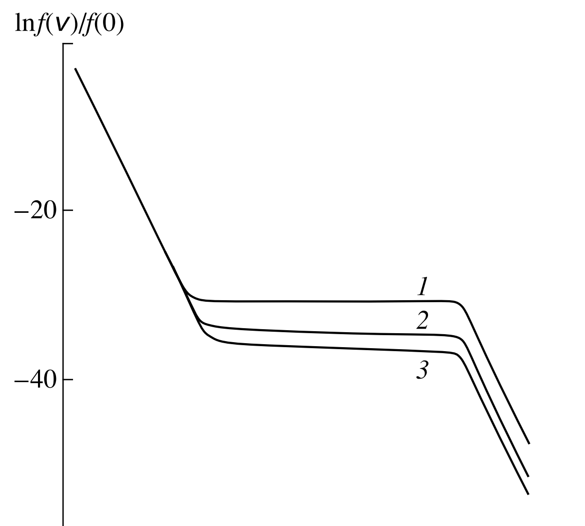

Fig. 7. Steady-state nonequilibrium distribution function vs. squared velocity. The computations were carried out for an energy flux with the constant intensity I = 0.01, for the source and sink functions in the form of δ functions (14), and for v_- = 4 and v_+ = 8. Curves *1*, *2*, and *3* were calculated from the Landau equation with the powers β = 1, 2, and 4, respectively.

The above analysis shows that, in a semiconductor plasmas, the power-law distribution function corresponding to a constant energy or particle flux in momentum space can exist in the energy range $(E - E_F) > E_F$. This distribution is formed by both collisions with electrons in the energy range $(E - E_F) > E_F$ and collisions with background (equilibrium) electrons.

As was shown above, the nonequilibrium electron distribution is close to a universal distribution if the intensity of the flux produced by the source and sink in momentum space is sufficiently high.

As an example, we consider a solid-state plasma that is affected by a flux of fast ions (with speeds higher than those of atomic electrons). A typical situation in a single ion track can be described as follows. Let the energy of a helium ion be $\varepsilon_i \approx 5$ MeV, the excitation potential be $\Phi \approx 100$ eV, and the ion mean free path in the target material be $R_{\rm tr} \approx 10^{-3}$ cm. The number of electrons produced during ionization processes along the path of a helium ion is about 10^4–10^5. The radius of the ionization track of an ion is comparable with the mean free path of the electrons produced ($R_e \approx 10^{-6}$ cm). The electron density in an ionization track is $n_{en} \approx 10^{19}$–10^{20} cm^{-3}, the equilibrium electron density being $n_e \approx 10^{22}$ cm^{-3}. The above analysis shows that, for such ratios of the electron densities, the source and sink are sufficiently intense for the nonequilibrium distribution function to form.

Let us now consider the mechanisms by which a fast ion loses its energy in a solid-state plasma. Note that, under conditions typical of the ion beam–driven inertial confinement fusion, the ion energy loss is independent of the ion density in the beam (cf. [19]). Taking into account this circumstance, we can calculate the energy lost by an ion from the Bethe–Bloch formula. According to [20], the energy of a fast nonrelativistic particle is transferred to a medium by the following two mechanisms. A fraction of the particle energy is expended on the excitation of collective oscillations associated with wake charge density waves, and the remaining fraction is transferred to individual electrons, with a subsequent ionization of the medium. The former fraction corresponds to the macroscopic energy loss in long-range collisions with little momentum transfer, and the latter fraction corresponds to collisions with large momentum transferred. The fraction of the particle energy that is lost through the excitation of collective oscillations is relatively large. The energy $\Delta\varepsilon_k$ converted into the wake charge density waves can be represented as

$$\frac{\Delta\varepsilon_k}{\Delta\varepsilon} = \frac{\ln(v/10v_0)}{2\ln(v/v_0)}, \qquad (17)$$

where $\Delta\varepsilon$ is the total energy lost by an ion and v_0 is the electron speed in the ground state of the hydrogen atom.

From representation (17), we can see that the energy $\Delta\varepsilon_k$ of the wake charge density waves is on the order of the total energy transferred from a particle to the medium.

A fast particle passing through the medium produces slow electrons by two equiprobable mechanisms: by avalanche ionization and by ionization through the excitation of plasma oscillations. The main features of the electron production through ionization by plasma oscillations are associated with long time and spatial scales of the wake charge density waves. Because of the long time scale of the waves, the secondary ionization inside the beam proceeds for a long time after the passage of a beam particle. A substantial number of slow electrons are liberated in cascade ionization caused by a high-energy secondary electron. Since the mean free path of such an electron in a medium is long, most of the slow electrons are produced in cascade ionization along its path. As a result, ionization by the wake field is the dominant process affecting the distribution of liberated electrons near the axis of the particle track, whereas cascade ionization is responsible for the distribution of liberated electrons at distances from the track axis that are on the order of the electron mean free path. Since a charged particle ionizes the medium by its self-field only during its passage and since the wake charge density waves play the role of a linear source of secondary electrons for a long time after the passage of the ionizing particle, the ionization dynamics is governed completely by the secondary electrons.

As was mentioned above, despite the short time required for an ion to pass along the track, the characteristic time of the avalanche ionization by wake charge density waves is fairly long (about 10^{-13} s).

In the interaction with a solid-state plasma, intense electromagnetic radiation with a frequency satisfying the condition $\omega \gg k_BT$ produces a large number of high-energy electrons, which, according to the above analysis, obey a steady-state nonequilibrium distribution function. Hence, in the case of a plasma irradiated by intense electromagnetic radiation or affected by the fluxes of fast particles, we deal with a nonequilibrium electron distribution, which differs from an equilibrium distribution in that it involves a large number of high-energy electrons. Let us analyze some physical consequences of such a fundamental change of the electron distribution function.

The density of the electron emission current from a solid-state plasma has the form

$$j = B_1 \int_{\varphi + eE_F}^{E_+} Ef(E)dE.$$

From this expression, we can see that, in a plasma with a nonequilibrium electron distribution, the electron emission current density is anomalously high, because, in the inertial interval, the distribution function decreases very gradually. The conduction characteristics of the medium are governed by the density of the current carriers, so that, in a semiconductor plasma with a nonequilibrium electron distribution, this density is very high, in contrast to the case of an exponentially decreasing equilibrium distribution function. That is why, under the action of intense fluxes of electromagnetic radiation or fast particles, the emission and conduction properties of a semiconductor plasma can become anomalous. Note that such anomalies have already been observed in some experiments (see, e.g., [4, 7]).

5. CONCLUSION

In conclusion, we summarize the results obtained from numerical modeling of the formation of steady-state nonequilibrium distribution functions.

For a distribution function computed from the kinetic equation with the Landau collision integral, we have established the following:

(i) In the interval between the source and sink located at certain positions in momentum space, the particles interacting through the Coulomb potential relax to a steady-state nonequilibrium distribution. Moreover, above certain source and sink intensities, the distribution function over almost the entire interval between the source and sink is of a universal nature; i.e., it obeys the same power law.

(ii) The functional dependence of the steady-state nonequilibrium electron distribution is sensitive nether to the extent to which the source and sink are localized in momentum space nor to the length of the inertial interval (i.e., the interval between the source and sink). It is, however, necessary that the positions of the source and sink be consistent with the direction of the energy flux in momentum space. Recall that the flux direction is determined by the nature of the interaction between particles and is entirely independent of the source and sink structures. In other words, the particles relax to universal local distribution functions.

(iii) For Coulomb and dipole interactions between Maxwellian molecules, the power s of the nonequilibrium distribution function that forms in the inertial interval decreases somewhat as the power β increases, in which case the values of the distribution function also become smaller.

(iv) For the source and sink intensities in the interval $0.001 \le I \le 0.1$, the distribution function computed from the Fokker–Planck equation is proportional to the intensity; as the intensity increases from 1 to 10, the values of the distribution function increase only by a factor of 3.

The fundamental change in the electron distribution function under nonequilibrium conditions leads to an anomalous enhancement of the conduction and emission characteristics of the medium. The results obtained in our investigations can be used to predict the behavior of semiconductors with an intrinsic or extrinsic conductivity in the fluxes of fast particles, as well as in intense electromagnetic radiation.

REFERENCES

1. B. A. Trubnikov, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1963), Vol. 1.
2. V. P. Silin, *Introduction to the Kinetic Theory of Gases* (Nauka, Moscow, 1971).
3. V. I. Karas', Pis'ma Zh. Tekh. Fiz. **1**, 1020 (1975) [Sov. Tech. Phys. Lett. **1**, 438 (1975)].
4. A. S. Aseevskaya, E. B. Ivkin, and B. G. Kolomiets, Pis'ma Zh. Tekh. Fiz. **2**, 305 (1976) [Sov. Tech. Phys. Lett. **2**, 117 (1976)].
5. W. L. Knecht, Appl. Phys. Lett. **6**, 96 (1965).
6. W. L. Knecht, Appl. Phys. Lett. **8**, 254 (1966).
7. S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, *High-Power Radiation Action on Metals* (Nauka, Moscow, 1970).
8. V. I. Karas', S. S. Moiseev, and V. E. Novikov, in *Proceedings of the XV International Conference on Phenomena in Ionized Gases, Minsk, 1981*, Vol. 1, p. 73.
9. A. N. Kolmogorov, Dokl. Akad. Nauk SSSR **30**, 299 (1941).
10. V. E. Zakharov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 35 (1965).
11. A. V. Kats, V. M. Kontorovich, S. S. Moiseev, and V. E. Novikov, Pis'ma Zh. Éksp. Teor. Fiz. **21**, 13 (1975) [JETP Lett. **21**, 5 (1975)].
12. V. I. Karas', S. S. Moiseev, and V. E. Novikov, Pis'ma Zh. Éksp. Teor. Fiz. **21**, 525 (1975) [JETP Lett. **21**, 245 (1975)].
13. V. I. Karas', S. S. Moiseev, and V. E. Novikov, Zh. Éksp. Teor. Fiz. **71**, 1421 (1976) [Sov. Phys. JETP **44**, 744 (1976)].
14. V. I. Karas', S. S. Moiseev, and A. P. Shuklin, Ukr. Fiz. Zh. **25**, 820 (1980).
15. A. V. Bobylev, I. F. Potapenko, and V. A. Chuyanov, Zh. Vychisl. Mat. Mat. Fiz. **20**, 993 (1980).
16. I. F. Potapenko, A. V. Bobylev, C. A. Azevedo, and S. S. Assis, Phys. Rev. E **56**, 7159 (1997).
17. I. F. Potapenko and V. A. Chuyanov, Zh. Vychisl. Mat. Mat. Fiz. **19**, 458 (1979).
18. J. M. Ziman, *Electrons and Phonons* (Clarendon, Oxford, 1960; Inostrannaya Literatura, Moscow, 1962).
19. M. M. Basko and M. V. Sokolovskiĭ, Preprint No. 177 (Inst. of Theoretical and Experimental Physics, Moscow, 1980).
20. Y.-H. Ohtsuki, *Charged Beam Interaction with Solids* (Taylor and Francis, London, 1983; Mir, Moscow, 1985).

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