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Quasi-Steady-State Particle Distributions for an Equation of the Landau–Fokker–Planck Type with Sources

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Abstract—The formation of nonequilibrium particle distributions for the power-law interaction potentials $U = \alpha/r^{\beta}$, where $1 \le \beta < 4$, is considered. The analytical and numerical studies are based on a one-dimensional (in the velocity space) kinetic equation of the Landau–Fokker–Planck type with energy (particle) sources (sinks) localized in the high-energy range. Fully conservative finite-difference schemes are used for numerical modeling. The resulting asymptotic estimates are confirmed by numerical computations. The results can be used to predict the behavior of intrinsic and extrinsic semiconductors influenced by particle fluxes or electromagnetic radiation.

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1. INTRODUCTION AND STATEMENT OF THE PROBLEM

The interest in nonequilibrium states of various physical systems is steadily increasing at present motivated by the development and wide use of high-power particle and energy sources. Quasi-steady-state nonequilibrium states of particle systems can be investigated by solving kinetic equations with particle sources and sinks. The existence of power-law energy distributions of particles was first shown theoretically in [1–4] and experimentally in [5, 6]. In [1, 2], similarity transformations were used to show that the Boltzmann kinetic equation for spatially homogeneous systems has exact power-law steady-state solutions. In [3, 4], direct evaluations of the Boltzmann and Landau collision integral showed that the power-law distributions $f(v) \sim v^s$, where s is the exponent, are steady-state solutions to kinetic equations that convert particle or energy fluxes in the phase space into nonzero constants. Such states of particles are similar to Kolmogorov wave spectra in turbulence. Moreover, these states depend only on the integral characteristics of the source and sink (see [4]). In particular, nonequilibrium spatially homogeneous systems include the electron subsystem in metals subject to ionization caused by alpha particles moving through the substance (see [5]). In [1–4], power-law solutions to the Boltzmann kinetic equation were obtained in which s depended only on the degree of homogeneity of the particle interaction potential. The dispersion properties of plasmas with power-law electron distributions and ionization equilibrium in such nonequilibrium steady-state states were also analyzed in [1–4]. It was shown in [6, 7] that the presence of two components (equilibrium and nonequilibrium) in the distribution function gives rise to plasma oscillations with linear dispersion, which are important, in particular, for the plasmon mechanism of superconductivity.

In this work, we analyze the formation of quasi-steady-state nonequilibrium distribution functions for a spatially homogeneous isotropic plasma of identical particles in the presence of localized particle (energy) sources (sinks) in the velocity space. The existence of nonequilibrium distribution functions assumes that sources and sinks of particles or energy are present in the momentum space. Such sources and sinks can be ensured by ion beams, high-power laser radiation, emission currents, fluxes of charged particles released in fusion or fission reactions, etc. Our study is based on an equation of the Landau–Fokker–Planck (LFP) type that models the Boltzmann equation with arbitrary interaction potentials (see [8, 9]). The asymptotic behavior of the solution to the LFP equation with weak sources (sinks) localized in the high-energy range is analyzed in Section 2. The analytical analysis is given at a physical level of rigor.

The evolution of nonequilibrium distribution functions and their dependence on various input parameters, such as the intensity of the sources and the degree of their localization in the velocity space, etc., are numerically studied in Section 3. The results are compared with experimental data obtained for a semiconductor irradiated with fast ions (see [10]). Fully conservative finite-difference schemes are used for the numerical modeling [8, 11]. The results are illustrated by plots. Section 4 presents concluding remarks.

In this paper, we use an LFP-type collision operator $\hat{I}[f, f]$ that models the Boltzmann equation [8] for the distribution function $f(\mathbf{v}, t)$:

$$\frac{\partial f}{\partial t} = \hat{J}[f, f] = \int d\mathbf{w} d\mu d\phi u\sigma(u, \mu)[f(\mathbf{v}')f(\mathbf{w}') - f(\mathbf{v})f(\mathbf{w})], \quad t \geq 0.$$

The integro-differential operator $\hat{I}[f, f]$ approximates $\hat{J}[f, f]$ for arbitrary interaction potentials when the actual gas consisting of particles with the cross section $\sigma(u, \cos\theta)$, where $u > 0$ is the relative velocity, is compared to a gas consisting of particles with another cross section: $\tilde{\sigma}(u, \cos\theta) = 0$ for $\theta > \theta_0$, where $\theta_0 \ll 1$ is a small scattering angle:

$$\frac{\partial f}{\partial t} = \hat{I}[f, f] = \frac{1}{8}\frac{\partial}{\partial v_i}\left\{\int d\mathbf{w} u\sigma(u)(u^2\delta_{ij} - u_i u_j)\left(\frac{\partial}{\partial v_j} - \frac{\partial}{\partial w_j}\right)f(\mathbf{v})f(\mathbf{w})\right\}. \tag{1}$$

In this model, the moments of the exact and model collision integrals coincide up to the third-order tensor moment and the fourth scalar moment. The number of particles and energy are conserved and the Boltzmann H-theorem holds. The model collision integral provides a well-defined description of the equations for Grad's 20-moment approximation. Finally, the exact solution to the Landau-type equation for Maxwellian molecules is the exact solution to the Boltzmann equation. Nevertheless, the natural applicability domain for the Landau-type operator is gases with the power-law interaction potential $U = \alpha/r^s$, where $1 \leq \beta < 4$, because the Boltzmann equation with such potentials has smoothing properties.

2. LANDAU-TYPE EQUATION WITH WEAK SOURCES LOCALIZED IN THE HIGH-ENERGY RANGE

Before proceeding to a numerical study, we analyze the key aspects of the problem by taking the asymptotic approach. The problem is solved under simplifying assumptions, and its solution will be used as a test in the numerical solution of more complicated problems.

For an isotropic distribution function, collision integral (1) has the form

$$\hat{I}[f, f] = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\frac{1}{v}\int_0^\infty dw Q(v, w)\left[wf(w)\frac{\partial f(v)}{\partial v} - vf(v)\frac{\partial f(w)}{\partial w}\right]\right\}, \tag{2}$$

where $Q(v, w)$ is the symmetric kernel given by

$$Q(v, w) = \frac{\pi}{8}v^3 w^3\int_{-1}^{1} d\mu(1 - \mu^2)u\sigma(u), \quad u^2 = v^2 + w^2 - 2vw\mu.$$

The particle density and energy expressed in energy units are defined as

$$n = (f, 1) = 4\pi\int_0^\infty dv v^2 f(v, t), \quad T = \frac{m}{3n}(f, v^2) = \frac{4\pi m}{3n}\int_0^\infty dv v^4 f(v, t), \quad t \geq 0, \tag{3}$$

where m is the electron mass.

Consider the kinetic equation with external sources (sinks) of energy (particles):

$$\frac{\partial f(v, t)}{\partial t} = \hat{I}[f, f] + \frac{n_S}{t_S}S(v), \quad f|_{t=0} = f_0(v); \quad 0 \leq v < \infty, \quad t \geq 0. \tag{4}$$

When $S(v) = 0$ (no sources or sinks), Eqs. (2)–(4) must be supplemented with the conservation laws for the number of particles $n = n_0 = \text{const}$ and for the energy $T = T_0 = \text{const}$. Note that, for some power-law functions ($f = C_1|v|^{p(\beta)}$) corresponding to interaction potentials of the form $U = C_2 r^{-\beta}$, the LFP-type collision integral (and the Boltzmann operator) vanishes, which can be checked by formal substitution. For Coulomb collisions, these functions with $p(1) = -5/2$ correspond to infinite energy in the complete velocity space. To ensure that integrals (3) are finite, the distribution function $f(v, t)$ is assumed to be bounded at $v = 0$ and to decay sufficiently rapidly as $v \longrightarrow \infty$. In this case, the only equilibrium solution to the problem is the Maxwell distribution.

In the general case, the sources and sinks can depend on time, the distribution function, etc. To reveal some general tendencies and to obtain a reference point for numerical computations, we restrict our consideration to time-independent sources of low intensity: $n_S/t_S = \varepsilon \ll 1$. A formal solution to Eq. (4) with small ε can be constructed in the form of the standard Chapman–Enskog expansion. For this purpose, we proceed to the new time variable $\tilde{t} = t\varepsilon$ and seek a solution to the equation $\partial f/\partial t = \hat{I}[f, f]\varepsilon^{-1} + S$ in the form of the series $f = f_0 + \varepsilon f_1 + \ldots$, where, as usual, $(f_i, 1) = (f_i, v^2) = 0$ for $i = 1, 2, \ldots$. Then,

$$f_0 = f_{\mathrm{Maxw}}(t) = n\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left(-\frac{mv^2}{2T}\right),$$

where n and T are determined by the equations

$$\frac{dn}{d\tilde{t}} = (S, 1), \quad n_{\tilde{t}=0} = n_0, \quad \frac{d}{d\tilde{t}}\left(\frac{3nT}{m}\right) = (S, v^2), \quad T_{\tilde{t}=0} = T_0, \tag{5}$$

which depend only on n_0 and T_0 but do not depend on a particular form of the initial function. The function f_1 is constructed by solving the inhomogeneous linearized equation

$$\partial f_0/\partial\tilde{t} = \hat{I}[f_0, f_1] + \hat{I}[f_1, f_0] + S.$$

The uniqueness of this solution is ensured by the orthogonality conditions. Similarly, we find f_i for $i \geq 2$. This method performs for all times rather than for the initial layer $\tilde{t} \sim \varepsilon$ alone, which is not of interest in this study.

Generally speaking, the Chapman–Enskog expansion becomes invalid in the high-energy range $v \gg v_{\mathrm{th}} = \sqrt{T/m}$, where the source is no longer small compared with the collision integral. Therefore, it becomes unclear how many expansion terms should be retained to obtain the leading asymptotic term for $\varepsilon \ll 1$ and $mv^2 \gg T_0$.

The typical time of Coulomb collisions in terms of the new variables $\tilde{t}_C$ is on the order of ε, and the characteristic time of the variations in the density and temperature are on the order of 1. It is then clear that there can exist an intermediate regime $\varepsilon \ll t \ll 1$ in which the relaxation has mainly ceased, while the density and temperature of the distribution have not varied substantially and the sources can still be assumed to be localized in the distribution tail.

It is well known that, at high velocities, the LFP equation simplifies to become a linear one:

$$\hat{I}[f, f] = \frac{n}{8}\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^3\sigma(v)\left(\frac{T}{m}\frac{\partial f}{\partial v} + vf\right)\right].$$

Here, we took into account that $Q(v, w) \longrightarrow \pi w^3 v^4\sigma(v)/6$ as $v \longrightarrow \infty$ (in the hot region). Moreover, for the power-law interaction potentials, we used $\sigma(u, \mu) = g_\beta(\mu)u^{-4/\beta}$ (see [12, 13]). Substituting this expression into (2) and letting $v \longrightarrow \infty$, we obtain the equation

$$\frac{\partial f}{\partial t} = \frac{ng_\beta}{8}\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^{3-4/\beta}\left(\frac{T}{m}\frac{\partial f}{\partial v} + vf\right)\right] + S, \tag{6}$$

where we used the notation

$$g_\beta = 2\pi\int_{-1}^{1} d\mu\, g_\beta(\mu)(1-\mu^2). \tag{7}$$

For example, in the case of the Coulomb potential with $\beta = 1$, formula (7) gives $g_1 = 32\pi e^4 L/m^2$. Then (6) yields the well-known linear Landau (Fokker–Planck) equation for plasma (see [14]).

Using the conventional renormalization procedure, we proceed to the following variables:

$$v' = v/v_{\rm th}; \quad t' = \tilde{t}/t_\beta, \quad t_\beta = \frac{32\pi v_{\rm th}^3}{n g_\beta} v_{\rm th}^{(\beta-1)/\beta}; \quad f' = \frac{4\pi v_{\rm th}^3 f}{n}, \quad S' = \frac{4\pi v_{\rm th}^3 t_S S}{n_S}.$$

By dropping the primes after the normalization and proceeding to the energy variable $x = v^2$, the LFP equation can be written as

$$\frac{\partial f(x,t)}{\partial t} = \frac{N}{x^{1/2}}\frac{\partial}{\partial x}\left\{x^\gamma\left[2\theta\frac{\partial f}{\partial x} + f(x)\right]\right\} + \varepsilon\frac{S(x)}{x^{1/2}}, \quad \gamma = 2\frac{\beta-1}{\beta}, \quad \varepsilon = \frac{n_S t_\beta}{n_0 t_S}. \tag{8}$$

The dimensionless density N and temperature θ,

$$N = \int_{-\infty}^{\infty} f x^{1/2} dx, \quad \theta = \frac{1}{3N}\int_{-\infty}^{\infty} f x^{3/2} dx,$$

vary with time according to

$$N_t = \varepsilon\int_{-\infty}^{\infty} S(x)dx, \quad (3N\theta)_t = \varepsilon\int_{-\infty}^{\infty} xS(x)dx; \quad N_0 = \theta_0 = 1. \tag{9}$$

The parameter values $\gamma = 0$, 1, and 3/2 correspond to the Coulomb and dipole interactions and to Maxwellian molecules, respectively.

Replacing f by $\varepsilon F(x)/N$ and t by τ/ε in Eq. (8) and letting $\varepsilon \longrightarrow 0$ gives the time-independent equation

$$\frac{\partial}{\partial x}\left[x^\gamma\left(2\theta\frac{\partial F}{\partial x} + F\right)\right] + S(x) = 0, \tag{10}$$

which includes the argument τ as a parameter (the argument of N_τ and θ_τ defined by Eq. (9)).

We have implicitly assumed thus far that $S(x)$ does not depend on ε. Now, we additionally assume that $S(x - x_1)$ is initially localized at the point

$$x_1 = 2\theta_0\left(\ln\frac{1}{\varepsilon} + y_1\right), \tag{11}$$

where y_1 is a (positive or negative) number independent of ε. Then, $S(x)$ is a compactly supported function (typically, $S(x) = \delta(x)$) normalized by the condition

$$\int_{-\infty}^{\infty} dx S(x) = 1. \tag{12}$$

Note that the Maxwellian distribution can be represented as

$$f_{\rm Maxw}(x) = \sqrt{\frac{2}{\pi}} N\theta^{-3/2}\exp\left\{-\frac{x}{2\theta}\right\} = \varepsilon\sqrt{\frac{2}{\pi}}N\theta^{-3/2}\exp\left\{-\left(\frac{x}{2\theta} - \ln\frac{1}{\varepsilon}\right)\right\} \equiv \varepsilon C\exp\{-y\}, \tag{13}$$

where $y = x/2\theta - \ln(1/\varepsilon)$.

Making another substitution $x = 2\theta z$, we rewrite Eq. (13) for the Coulomb interaction (i.e., for $\gamma = 0$):

$$\frac{d^2F}{dz^2} + \frac{dF}{dz} = -\frac{\theta}{\theta_0}S\left[\frac{\theta}{\theta_0}z - \ln\frac{1}{\varepsilon} - y_1\right]. \tag{14}$$

A unique solution can be obtained at the initial stage of the distribution tail formation, when $\tau \ll (\ln 1/\varepsilon)^{-1}$ and the macroscopic parameters have varied insignificantly: $N \simeq N_0$ and $\theta \simeq \theta_0$. A solution to Eq. (14) is constructed in the form $F(y) = \varphi[y - \ln(1/\varepsilon)]$. Then, we obtain the equation $\varphi'' + \varphi' = -S(y - y_1)$, which is considered for all $-\infty < y < \infty$. As before, $\varphi(x) \longrightarrow 0$ as $y \longrightarrow \infty$. The matching condition for Maxwellian distribution (13) can be uniquely specified as $\lim e^y\varphi(y) = C$ as $y \longrightarrow -\infty$. Finally, we obtain

$$\varphi(x) = e^{-x/(2\theta)} + \frac{1}{N}\int_{-\infty}^{x} dz e^{-(x-z)/(2\theta)} \int_{z}^{\infty} du S(u);$$

i.e., the solution in the tail is uniquely determined. This solution can be extended to several sources $S = \Sigma S(y - y_i)$ and to negative sources (sinks). Recall that this formula works as long as $\theta \approx \theta_0$, i.e., at rather short time intervals. However, if we return to the initial time units, the applicability condition for this solution becomes $1 \ll t \ll [\varepsilon\ln(1/\varepsilon)]^{-1}$, which is a rather long time interval. Remarkably, both the density and temperature remain virtually unchanged on this interval, but a new quasi-steady-state distribution is established in the high-energy range. Thus, we obtain an intermediate asymptotic representation, which is exact when $t \longrightarrow \infty$, $\varepsilon \longrightarrow 0$, and $x \longrightarrow \infty$, so

$$\Delta t = \varepsilon \ln\frac{1}{\varepsilon} t \longrightarrow 0, \quad y_1 = \left(\frac{x}{2\theta_0} - \ln\frac{1}{\varepsilon}\right) \text{ is finite.}$$

In numerical computations, this intermediate asymptotics will be observed when the sources in the distribution tail are located close (in order of magnitude) to the point $x(\varepsilon) \simeq 2\theta_0\ln(1/\varepsilon)$. The length of the interval Δt can be estimated from expressions (9) and (11).

In particular, if the source and sink are given by $S(x) = I_+\delta(x - x_+) - I_-\delta(x - x_-)$, then, for the Coulomb scattering with $\gamma = 0$, we obtain the solution

$$f(x, t) = C\exp\left[-\frac{x}{2\theta}\right] + \frac{\Delta(x)}{N}, \tag{15}$$

where

$$\Delta(x) = I_+\left\{\eta[x_+ - x] + \eta[x - x_+]\exp\left(-\frac{x - x_+}{2\theta}\right)\right\}$$

$$-I_-\left\{\eta[x_- - x] + \eta[x - x_-]\exp\left(-\frac{x - x_-}{2\theta}\right)\right\}.$$

Here, $\eta[y]$ is the unit function equal to 1 if $x > 0$ and equal to 0 if $x < 0$.

In the general case, when $\gamma > 0$, we obtain

$$f(x) = Ce^{-x/(2\theta)} + \frac{1}{N}\int_{x}^{\infty} S(z)dz \int_{-\infty}^{x} e^{-(x-z)/(2\theta)} z^{-\gamma} dz + \frac{1}{N}\int_{-\infty}^{x} S(z) \int_{-\infty}^{z} u^{-\gamma} e^{-(x-u)/(2\theta)} du dz. \tag{16}$$

For the dipole interaction ($\gamma = 1$), the integrals in (16) can be expressed in terms of the integral exponential function Ei(x) (or li(e^x)). For $\gamma = 3/2$, they are reduced to Dawson's integral.

Numerical computations confirm these analytical estimates, which will be demonstrated in the following section.

3. NUMERICAL STUDY OF THE EVOLUTION OF THE DISTRIBUTION FUNCTION IN THE PRESENCE OF EXTERNAL PUMPING

The numerical implementation of this problem faces a fundamental difficulty, namely, the nonlinearity of the collision integral. As was noted above, in the case of no external sources, two conservation laws must hold; otherwise, the dissipative properties of the scheme may distort the result by introducing spurious sinks or sources. For this reason, we used difference schemes that properly take into account the nonlinearity of the simulated equation [11].

For numerical modeling, we used nonlinear operator (2) with a symmetric kernel $Q(v, w)$ for the power-law interaction potentials:

$$Q(v, w) = \frac{a(v, w)(v + w)^{\eta + 4} + b(v, w)|v - w|^{\eta + 4}}{(\eta + 2)(\eta + 4)(\eta + 6)},$$

where

$$a(v, w) = [(\eta + 4)vw - (v^2 + w^2)], \quad b(v, w) = [(\eta + 4)vw + (v^2 + w^2)], \quad \eta = (\beta - 4)/\beta.$$

For charged particles, $\beta = 1(\eta = -3)$ and we have $Q(v, w) = (2/3)w^3$ for $w \le v$ and $Q(v, w) = (2/3)v^3$ for $w \ge v$.

To construct a finite-difference scheme, the infinite interval in the velocity space was replaced with the finite interval $[0, v_{max}]$, which takes into account the high-energy particles, and the boundary condition for the distribution function was $f(v_{max}, t) = 0$. The sources and sinks were specified as delta functions: $S_\pm \sim I_\pm\delta(v - v_\pm)/v^2$. For the intensities given by $I_+ = I_- v_-^2/v_+^2$, the energy supplied from the outside was zero and we had an analogue of a particle flux. Additionally, we considered the exponentially distributed sources (sinks)

$$S_\pm \sim I_\pm \exp[-b(v - v_\pm)^2] \tag{17}$$

in order to examine the dependence of the nonequilibrium distribution functions on the shape of the source. Strictly speaking, the introduction of negative sources (sinks) independent of the distribution function can be rather problematic. In this case, we can always set initial conditions such that the solution becomes negative in the sink field after a time. For this reason, the sinks are frequently simulated by terms corresponding to the particle absorption (proportional to the desired distribution function).

In the discrete case, the function $\delta(v - v_1)$ is nonzero only at $v = v_1$.

The initial distribution was specified as the Maxwellian distribution

$$f^0(v) = \frac{4}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{3/2}\exp\left(-\frac{3}{2}v^2\right), \quad N_0 = 1, \quad \theta_0 = \frac{1}{3},$$

or as a deltalike function. Note that the results virtually did not depend on the form of the initial distribution function, except for possibly at the initial time. At every time step, the function was iterated, the number of particles was conserved up to machine accuracy, and the energy was conserved to within 7–8 digits.

First, we consider the evolution of the distribution function for $\beta_{1, 2, 3} = 1, 2$, and 4 under the influence of only sources. The formation of a nonequilibrium distribution can be divided into three stages. At the first (short) stage, the system remembers the initial conditions. This period does not differ widely for different values of β and is equal approximately to $t \sim 1$. The basic part of the distribution is formed at the second stage. Its duration depends substantially on the location of the source v_+ but does not depend on its intensity. Depending on the source intensity, the distribution function becomes plateau-like or decreases slightly between the source and the cold domain. The establishment of a steady-state distribution completes with the formation of a distribution tail. This stage depends substantially on β.

Figure 1 shows steady-state nonequilibrium functions vs. the squared velocity (on a logarithmic scale). The computations were performed for $v_+ = 6$ and $I_+ = 10^{-6}$. The solid curve corresponds to $\beta = 1$; the dotted curve, to $\beta = 2$; and the dashed curve, to $\beta = 4$. The numerical results illustrate the dependence derived in the preceding section and agree well with the asymptotic solution. It can be seen that even relatively weak sources lead to a striking increase in the nonequilibrium portion of the distribution. In terms of our units, the basic part of the distribution ceases to evolve at $t \simeq 50$ for $\beta = 1$, at $t \simeq 2$ for $\beta = 2$, and at $t \simeq 1$ for $\beta = 4$. Figures 2 and 3 display the formation of the distribution tail for $\beta = 1$ and $\beta = 4$, respectively, which is characterized by the deviation of the distribution function φ from the Maxwellian distribution: $f = f_{Maxw}(1 + \varphi)$. The distribution tail is formed much as in the relaxation of a monochromatic beam. Since the plasma is heated insignificantly at the parameters used, the front $\varphi(t)$ is virtually not smeared. The results correspond to $t = 1, 10, 25, 50, 100$, and 150 in Fig. 2 and to $t = 0.05, 0.1, 0.15, 0.2, 0.25, 0.4$, and 0.5 in Fig. 3. The deviations $\varphi = f/f_{Maxw} - 1$ were calculated for $v_+ = 6$ and $I_+ = 10^{-6}$.

Figure 4 presents the distribution function generated under the influence of a (a) source and (b) sink localized at $v_+ = 6$ and $v_- = 3.5$, respectively, and normally distributed with $I_+ = I_- = 10^{-6}$ and $b = 10$ (see (17)). It is evident (Fig. 4a) that the shape of the source has an effect only in the close vicinity of v_+ but does not change the basic characteristics. To show the weak decrease in the distribution function, a portion of the graph is plotted on an enlarged scale. Due to the sink, the nonequilibrium part of the distribution function in the transition to its equilibrium part at v_- becomes sharper.

Figure 5 displays the steady-state distribution function for $\beta = 2$ vs. the squared velocity for a source located at $v_+ = 8$. The dotted curve corresponds to $I_+ = 10^{-5}$ and the dashed curve, to $I_+ = 10^{-8}$. The solid

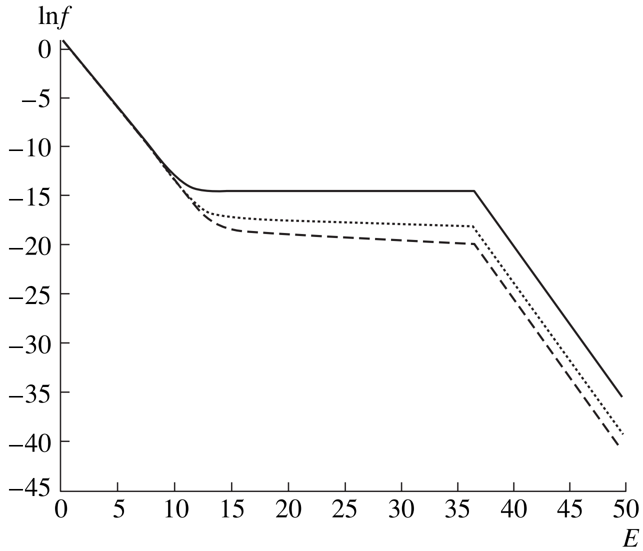

Fig. 1.

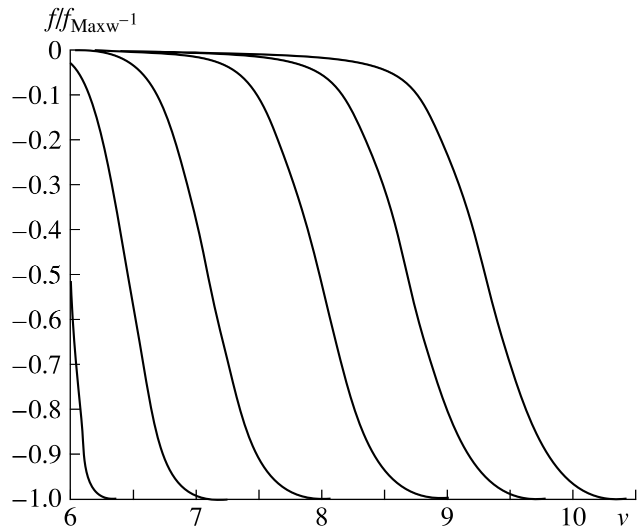

Fig. 2.

curve shows the distribution function for a source located at $v_+ = 5$ with $I_+ = 10^{-8}$. No sinks are present. It can be seen that the height of the plateau is proportional to the source intensity in agreement with the analytical result. For $I_+ = 10^{-8}$, the overheated population of the particle spectrum spreads to a colder region to the values $v \simeq 4$, and, for $I_+ = 10^{-5}$, it reaches $v \simeq 3$, which is fairly close to the basic part of the distribution. The slope of the distribution functions in the tail shows that the temperature does not vary. The transition period of the nonequilibrium distribution function does not depend on the length of the interval $\Delta v = v_+ - v_-$.

If the source and the sink are interchanged, the resulting nonequilibrium function has the form of an equilibrium distribution with a poorly populated tail starting at $v \geq v_-$, i.e., with a parallel shift in the high-velocity range.

Naturally, when the intensity of the sources increases noticeably, the heating of the plasma in the evolution increases as well (the temperature rises) and the nonequilibrium solutions become increasingly more "quasi-steady-state" rather than "steady-state."

The conductivity of the medium is determined by the density of the charge carriers. Therefore, we can assume that semiconductor plasmas with similar nonequilibrium distribution functions must possess anomalous conductivity and emissions exceeding the thermal emission by several orders of magnitude.

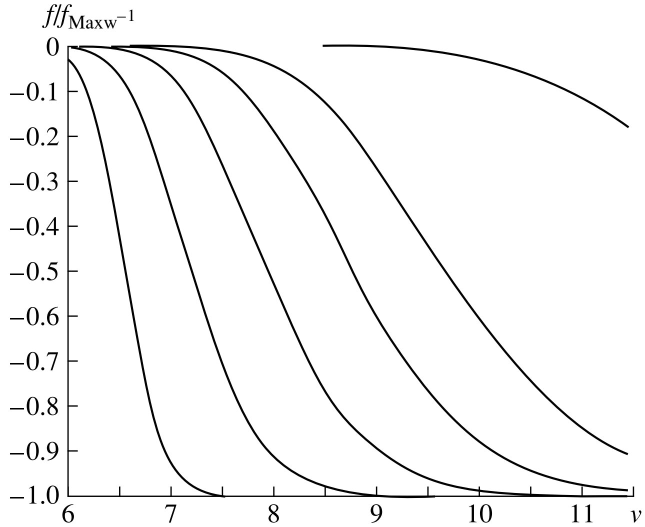

Fig. 3.

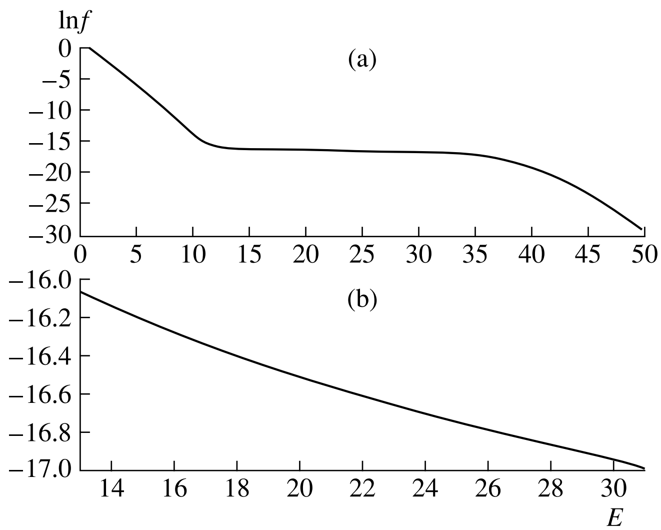

Fig. 4.

To conclude, we compare the numerical results with experimental data obtained for a thin semiconducting GaAs film irradiated with fast ions H^+ with energy of 1.25 MeV (see [10]). Since the proton free path is $l = 3 \times 10^{-6}$ m and the ion velocity is $v_{Ht} = 7.5 \times 10^6$ m/s, the time period within which the proton loses its basic energy can be calculated as $t_{tr} = 4 \times 10^{-13}$ s. For the given GaAs sample with the electron density $n_e = 5 \times 10^{24}$ m^{-3}, the typical velocity is $v_{th} = 6 \times 10^5$ m/s. Then the corresponding electron–electron relaxation time is estimated as $t_{ee} = 3 \times 10^{-14}$ s. This result suggests that, due to the presence of a flux in the velocity space, there is enough time for a nonequilibrium distribution function to be formed. The intensity of the sources arising due to ionization via direct electron collisions and due to plasma wave excitation is approximately equal to $I_+ \approx 3.3 \times 10^{37}$ m^{-3} s^{-1}. Therefore, in the computations, we should use a normalized source intensity on the order of 0.01–0.1 and a source action time on the order of 10. A source located at $v_{+,1} \approx 3.5$ corresponds to the excitation of a plasmon and at $v_{+,2} \approx 7$ correlates with the ionization caused by electron collisions. The main losses from the sample are caused by ion–electron emissions from the film surface. The electrons with energies higher than the work function A and the retarding potential U escape from the sample within the time τ_{ex}, since the emitting layer depth $d_{em} = v_{th}\tau_{ee}$ exceeds the electron free path λ in the semiconductor. The sink function is zero for energies lower than $\mathcal{E} = E_F + A \approx 5.65$ eV and is distributed in the

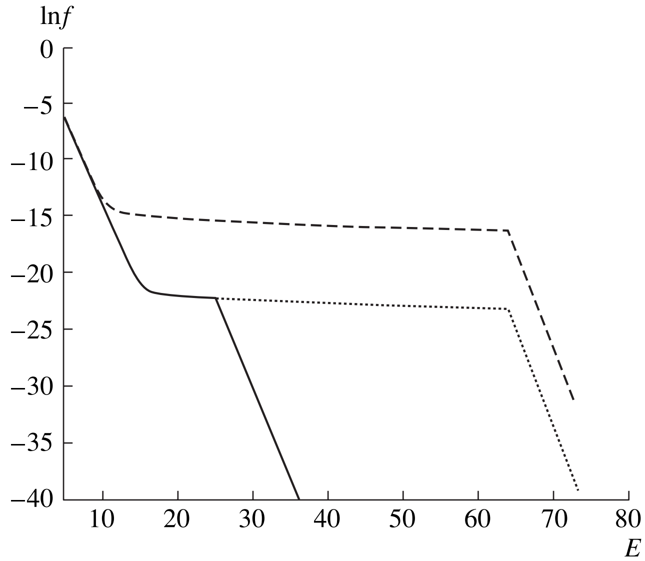

Fig. 5.

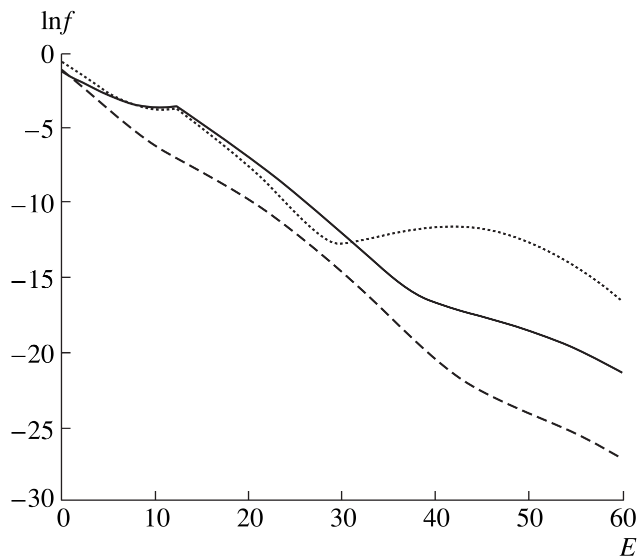

Fig. 6.

velocity space as follows:

$$S_- = \frac{\tau_{ee}}{\tau_{ex}} f\left(\frac{v}{v_{th}}\right) = \frac{\lambda v \tau_{ee}}{d_{em}^2} f\left(\frac{v}{v_{th}}\right) \approx 0.05 \frac{v}{v_{th}} f\left(\frac{v}{v_{th}}\right).$$

The evolution of the distribution function vs. the squared velocity is shown in Fig. 6. The dotted curve corresponds to $t = 15$; the solid curve, to $t = 20$; and the dashed curve, to $t = 30$. The source of intensity $I_{+,1} = 0.1$ was switched off at the point $v_{+,2} = 3.5$ at $t = 20$, and the source of intensity $I_{+,2} = 0.075$ was switched off at the point $v_{+,2} = 7$ at $t = 2$. A sink of intensity $I_- = 0.075$ was present in the range $v > 2$.

The dependence of the emission current on the retarding potential can be estimated using the formula

$$J = \text{const} \int_{E_F + U + A}^{\infty} dE f(E) E,$$

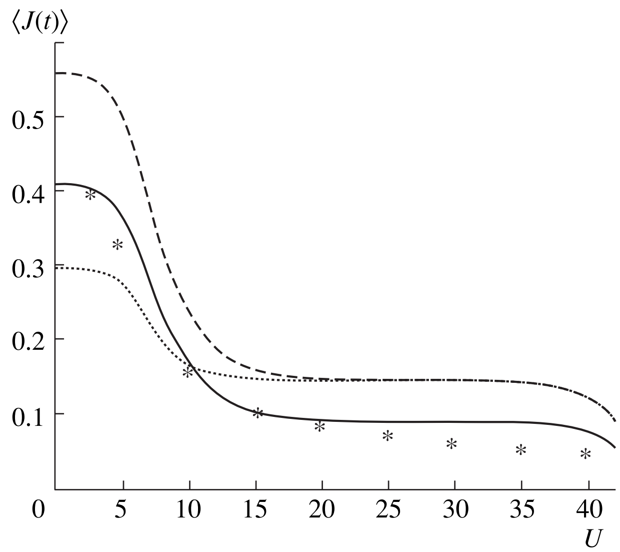

Fig. 7.

where E_F is the Fermi energy. For the experimental situation in question (see [10]), the ion beam currents do not exceed 10 μA. In this case, the emission current reflects the unsteadiness of the sources, since there is enough time for the electron function to pass all the formation stages on each ion track. Therefore, the experimentally observed dependence of the emission current on the retarding potential is a superposition of the currents existing at all the stages. Figure 7 shows the emission current as a function of the retarding potential: the dotted curve corresponds to the current averaged over $t = 10$; the dashed curve, over $t = 20$; the solid curve, over $t = 100$; and the stars show the experimental results from [10]. The main conclusion drawn from the comparison with the experiment data is that the unsteadiness of the sources is an important factor to take into account in the experimental data processing.

4. CONCLUSIONS

The formation of quasi-steady-state nonequilibrium distributions in the presence of energy (particle) sources (sinks) has been studied numerically on the basis of a nonlinear kinetic equation of the Landau (Fokker–Planck) type for weak long-range potentials $U \sim \alpha/r^{\beta}$, where $1 \le \beta < 4$. The evolution of the distribution function and of its structure was analyzed in detail. An intermediate asymptotic representation was analytically obtained for weak sources located in the high-energy range. The asymptotic analysis confirmed the high reliability and accuracy of the finite-difference schemes used. A new quasi-steady-state distribution was found to be established over a long time interval when the energy and density vary insignificantly. The nonequilibrium part of the distribution function can differ from the equilibrium by many orders of magnitude. This circumstance can play an important role in the creation of radioisotope current sources that have stimulated conductivity characteristics that significantly exceed those of the thermal emission sources of the current.

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